

# DYNAMIC TESTING AND SIMULATIONS OF GEARS AND GEARBOXES

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and

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# CENTRE INTERMECH - MO.RE.

HEAD FRANCESCO PELLICANO

Tecnopole of Modena



Tecnopole of Reggio Emilia



Interdepartmental Centre for Applied Research and Services in the  
Field of Advanced Mechanics and Engines

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Automation,  
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Design,  
Condition  
Monitoring and  
Drives for  
Mechatronics

Enertronics,  
Thermo-fluid  
dynamics and  
Industrial  
Sustainability

Rational Design of  
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Efficient Management  
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Processes

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Telecommunications

XILAB (X-in-the-  
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and Optimization  
Lab)

THERMAL  
FLUIDS AND  
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OptoPhemLab

Lab. Arc –  
Microman

Materials science and  
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implementation and  
characterization of advanced  
materials

Fluid Power and  
Motors (OIMo)

Lab of  
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Powertrain

Lab Tension  
Analysis

Lab. Integrated  
Design and  
Simulation  
(LAPIS)

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# THE VIBRATION AND POWERTRAIN LAB.

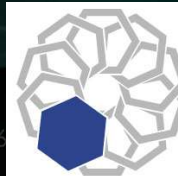


**University of Modena and Reggio Emilia**

**Department of Engineering "Enzo Ferrari"**

**InterMech Mo.Re. Centre**

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**TECNOPOLO MODENA**





## PEOPLE:



Head of Centre InterMech MO.RE.

**Prof. Francesco Pellicano**



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website



Researcher  
**Dr. Antonio Zippo**



Visiting professor  
**Prof. Farhad S. Samani**



Researcher  
**Dr. Giovanni Iariccio**



Ph.D. student  
**Moslem Molaie**



Ph.D. student  
**Razieh Ebrahimnejad**

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## Lab. Activities

### Scientific Research

- European projects
- International cooperations
- Gear, Vibration
- Nanoscience, Materials
- Bio-engineering

### Industrial Projects

- FP7 and H2020 programs
- Gear design, simulation and testing
- Vibration analysis and testing
- Development of new theories for thin-walled structures: SPACE APPLICATIONS
- **Service:**
  - Dynamic tests on shaking table
  - Modal analysis (numerical and experimental)
  - Vibration measurements
  - Complex gear train analyses

### Education

- International student exchange
- International visiting Professor exchange
- Lab experience for students
- Courses:
  - ✓ Mechanical Vibrations
  - ✓ Multibody Dynamics
  - ✓ Mechanics of Vehicle



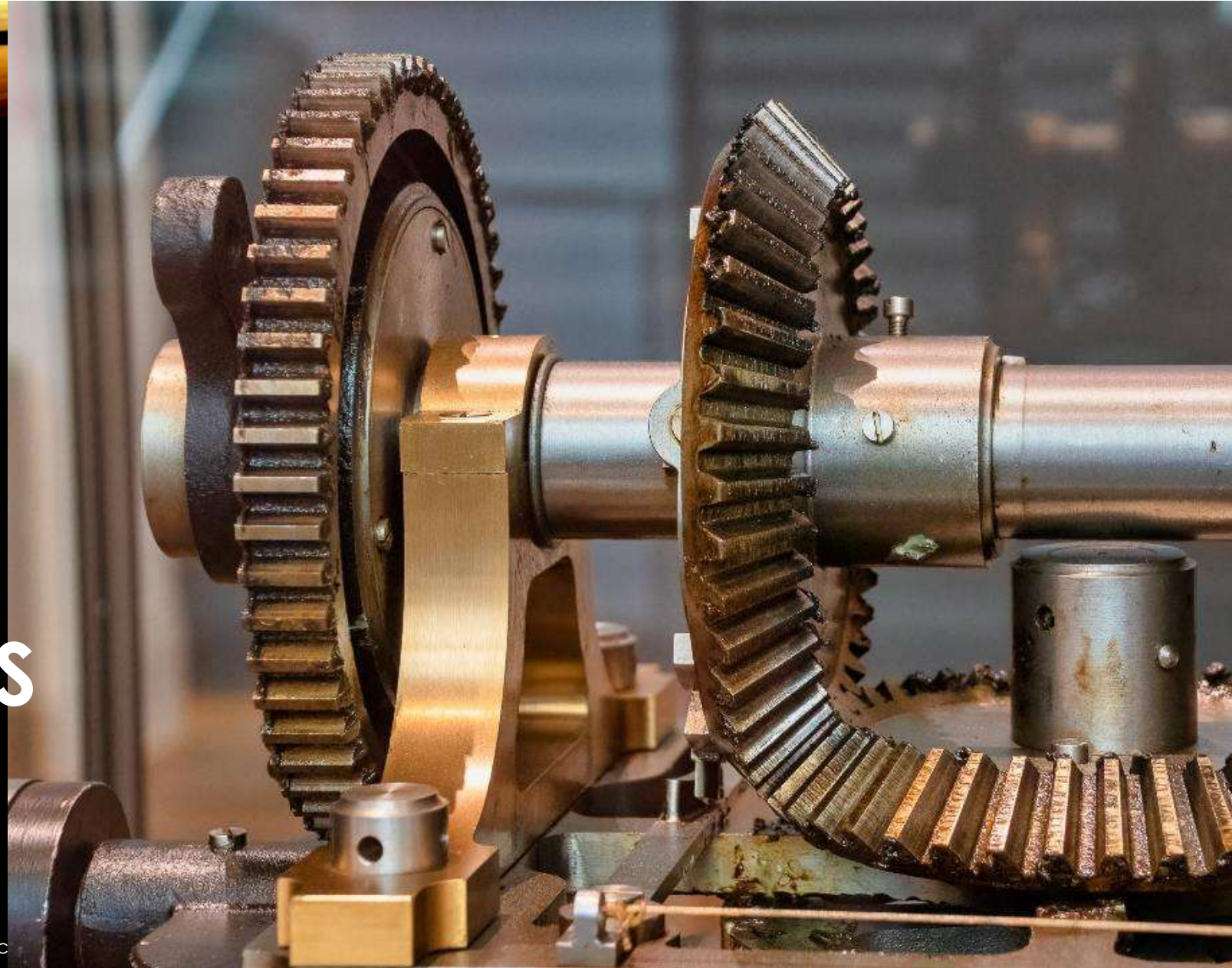
# INTERNATIONAL COLLABORATIONS



- America (USA, Canada, Brazil)
- Europa (UK, France,...)
- East-Europe/Asia (Ukraine, Russia, ...)
- Middle East (Iran)
- Oceania (New Zealand)
- Waitako University (Hamilton)
- Semenov Institute (Before:2022, Moscow)
- Gear Lab, OHIO State University (USA)
- Loughborough University (UK)
- McGill University (CA)
- INSA Lyon
- KPI University (Kharkov)
- Kerman University (Iran)

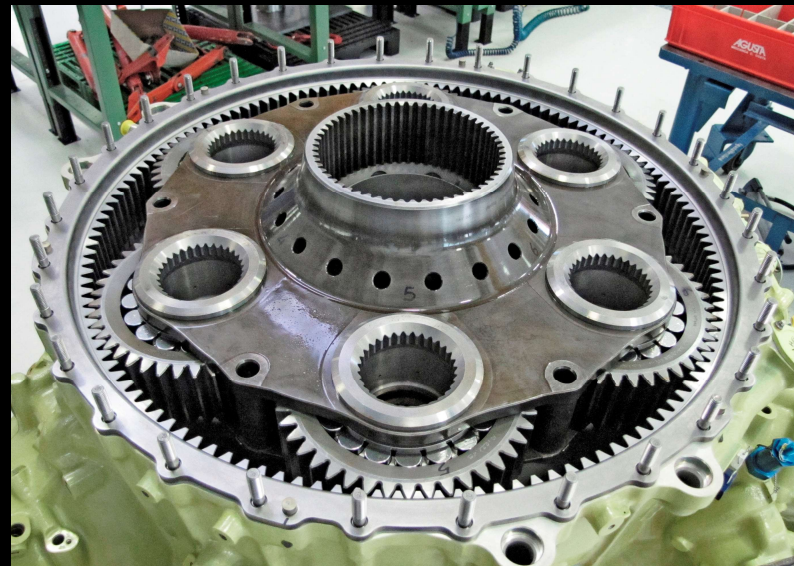


# GEARS and GEARBOXES



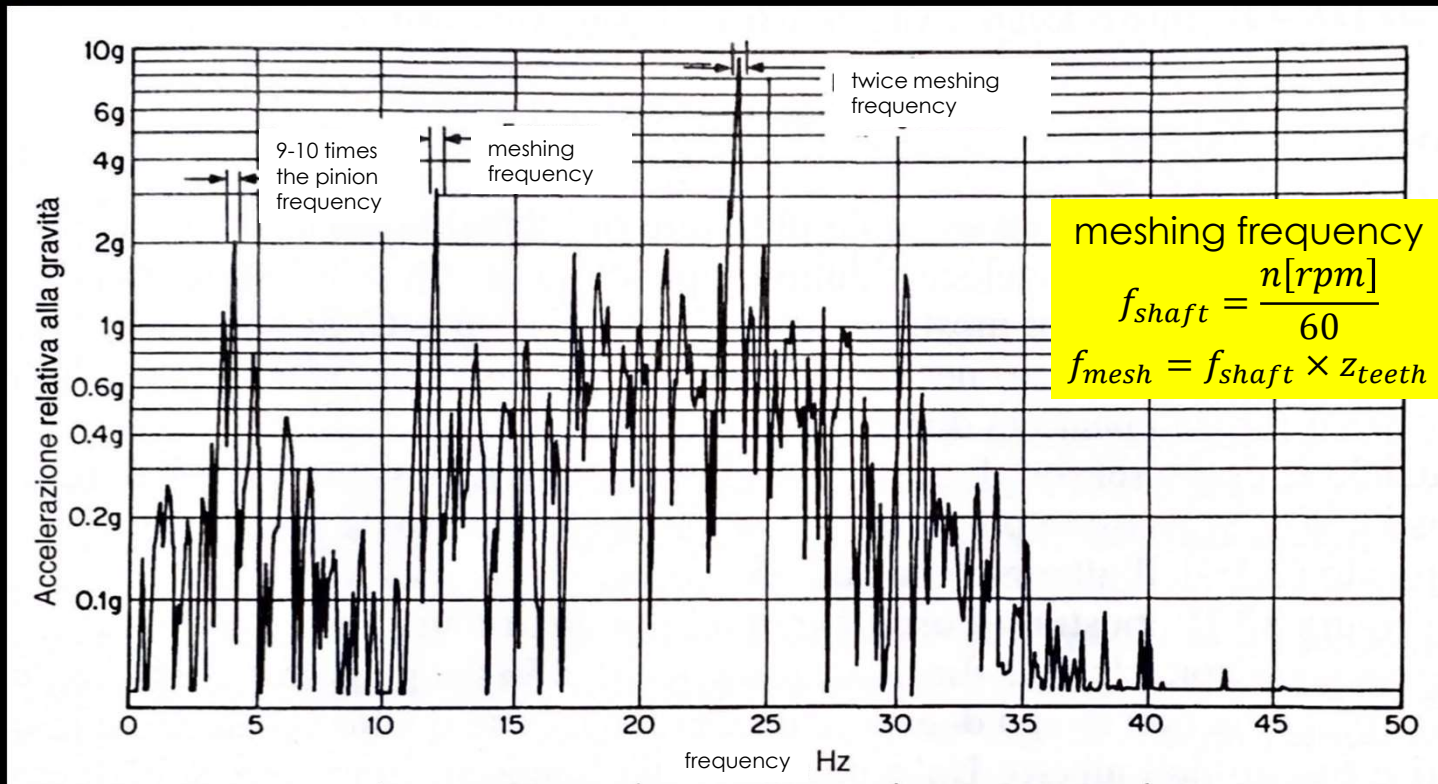
F. Pellicano - A. Zippo - francesco.pellicano@unipi.it

# From the Unimore Course for Engineers: Design and simulation of gears and transmissions: NVH issues





## Example of vibration spectrum of a gearbox

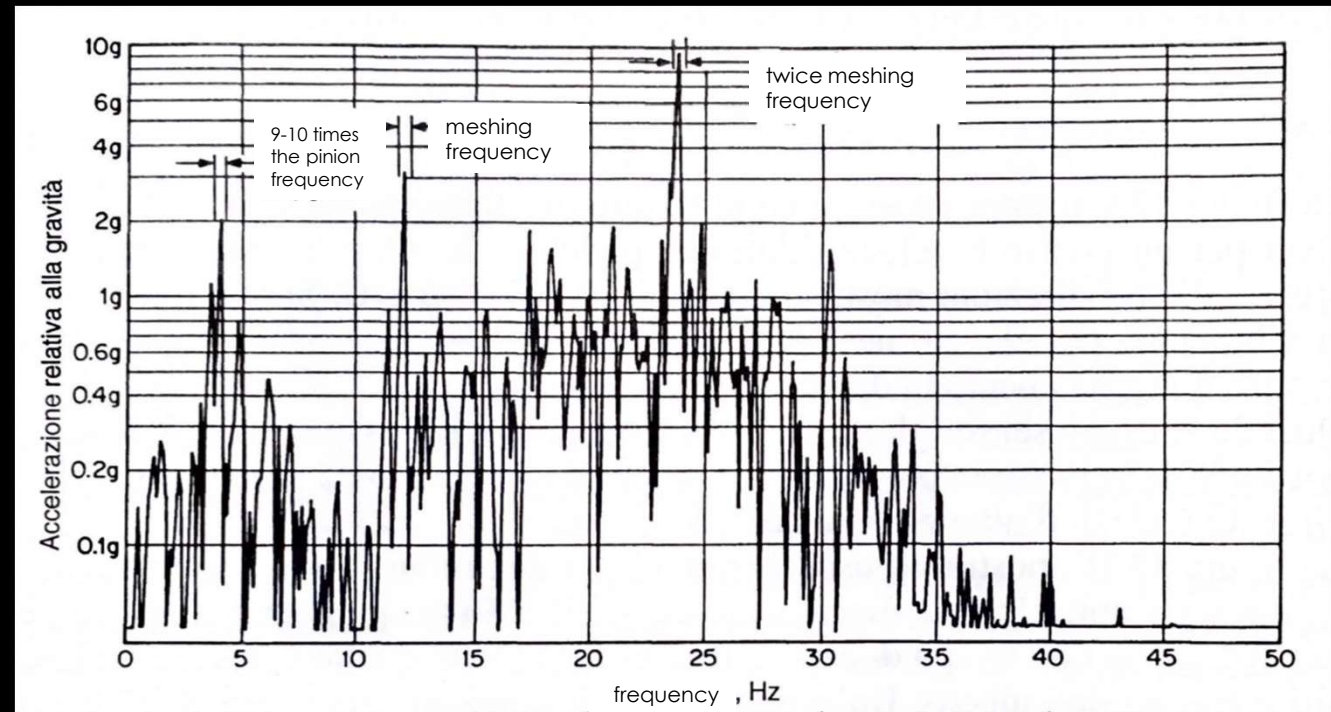


This gearbox had a failure due to wear, but the vibrations were not high



## Example of vibration spectrum of a gearbox

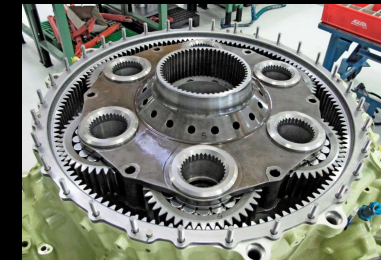
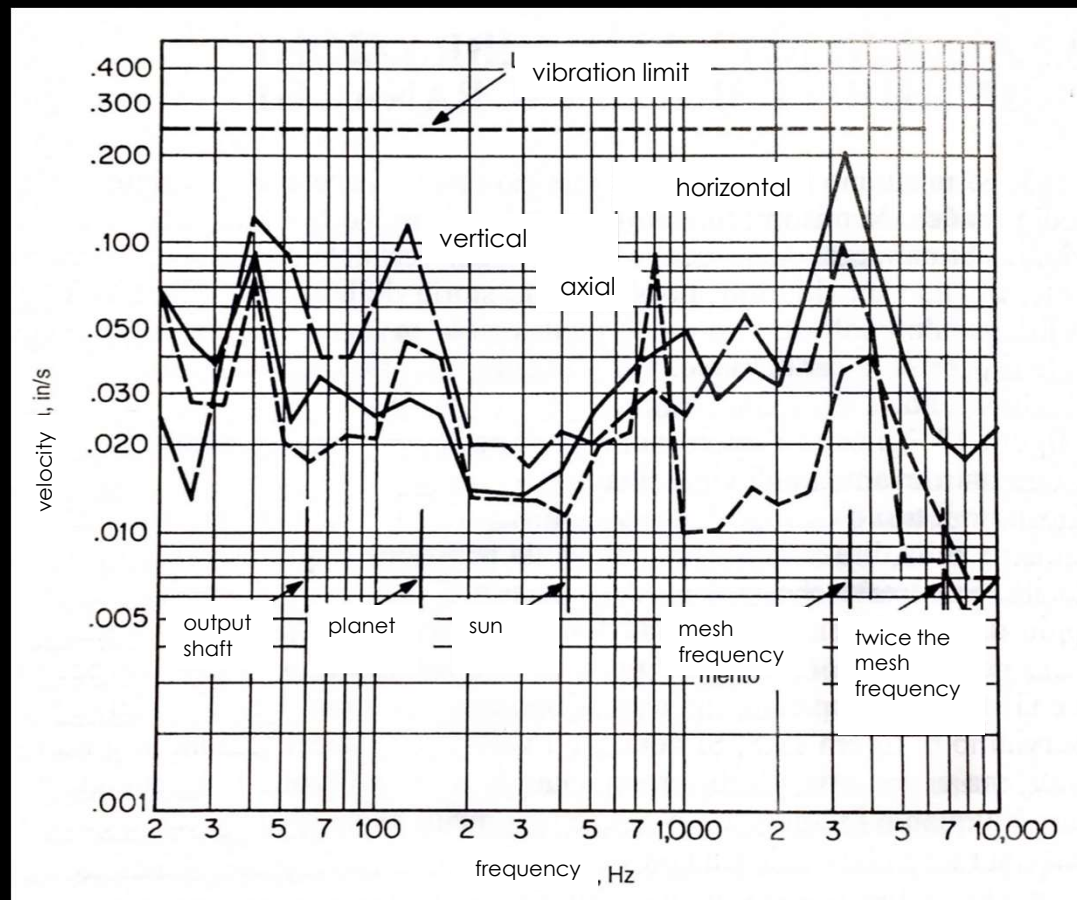
- Normally measurements are carried out on different points with triaxial accelerometers
- Peaks at the meshing frequency and its double
- A peak at 9 times the pinion frequency
- There was still wear on 9 teeth (9 bumps)



- The accelerations themselves do not appear worrying as a level
- The peak at  $9 \times$  pinion frequency is disturbing (Dudley)
- There is no reason for the presence of this peak except the damage

## Example of vibration spectrum of a planetary gearbox

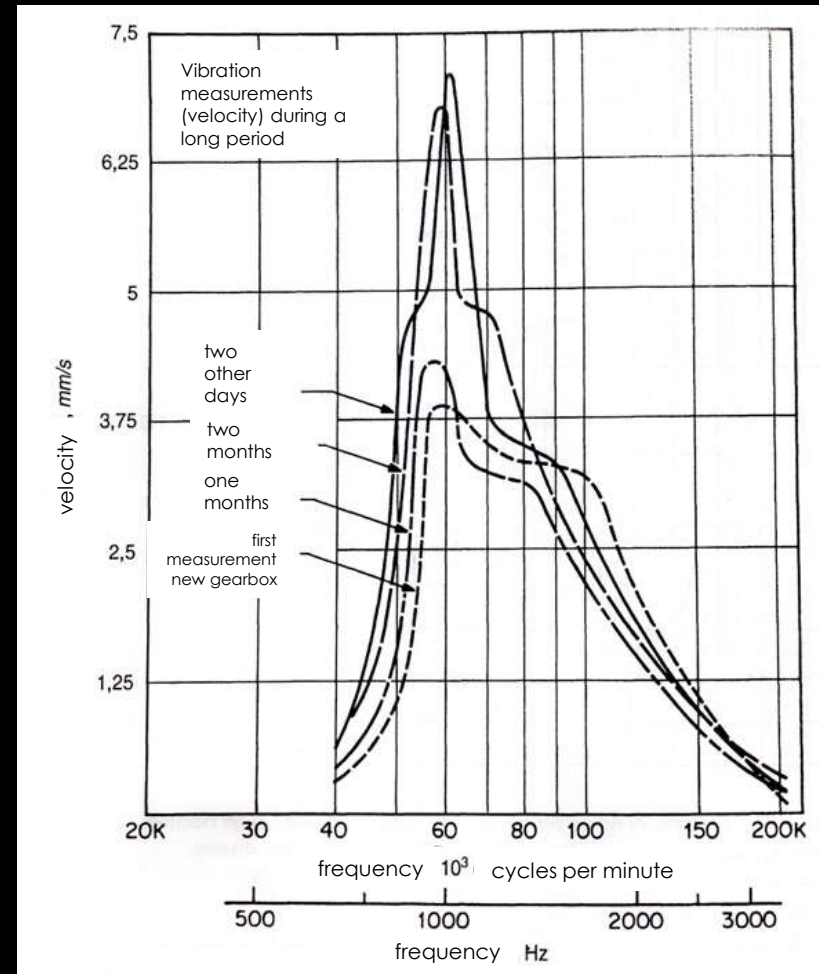
- Proximity sensor measurements
- The maximum is at the meshing frequency
- At twice the mesh frequency there is nothing
- Interesting the different character of the various spectra



$$\omega_{mesh} = \omega_{sun} \frac{Z_{sun} Z_{ring}}{(Z_{sun} + Z_{ring})}$$

## Example of vibration spectrum of a gearbox with failures

- Effect of progressive failures
- As soon as it was put into service, the first survey was carried out and the spectrum was recorded.
- A progressive increase in vibrations was observed.
- Given the sudden increase in vibrations, it is decided to take it out of service





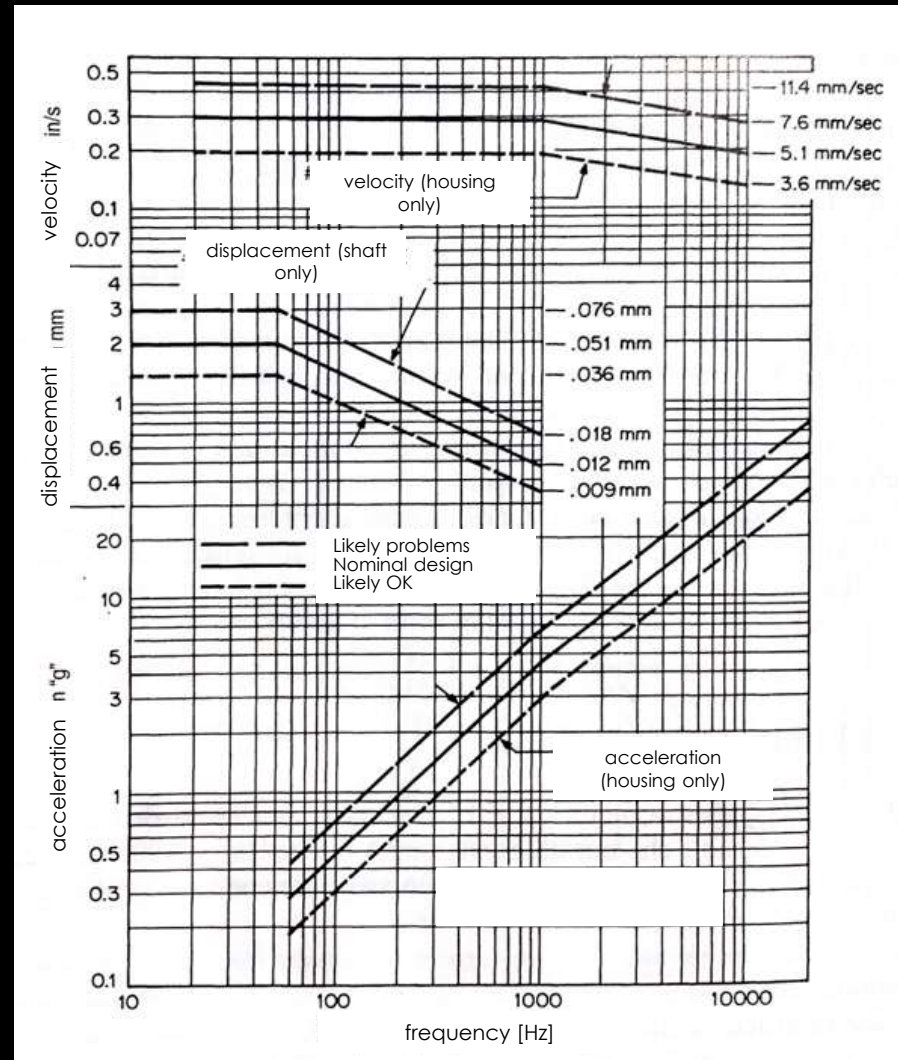
### VIBRATIONAL LIMITS

- Gear applications range from nano to macro scale, with powers ranging from fractions of W to thousands of kW
- Vibration limits are necessarily variable
- The type of gear (e.g. straight or helical teeth) greatly influences the levels
- The gear material influences levels as well as treatments and machining
- The material of the housing influences the outer vibration levels (cast iron dampens more than steel, aluminium dampens less)

## VIBRATIONAL LIMITS

- Normogram suggested by Dudley
- Top curve: suspected problems
- Continuous curve: smooth operation
- Dashed curve: tightly controlled gear

Note: Displacement measurements lose meaning at high speeds because sensors cannot dynamically measure much below microns



## VIBRATIONAL LIMITS

Factors influencing the vibration limits

Factor	Increases the vibration limits	Reduces the vibration limits
Durability (full power)	Less than 2000h	More than 20000h
Material hardness	38HRC or less (hardened steel)	50-65 HRC (case-hardened)
Type of gear	Spur or bevel gear	Helical or Spiral bevel gear
Accuracy of teeth	Average gears (machined)	High accuracy (grinding)
Peripheral primitive speed	Less than 25.4 m/s	More than 25.4 m/s
Max power	Less than 400kW	More than 1500kW
Weight	Light	Heavy

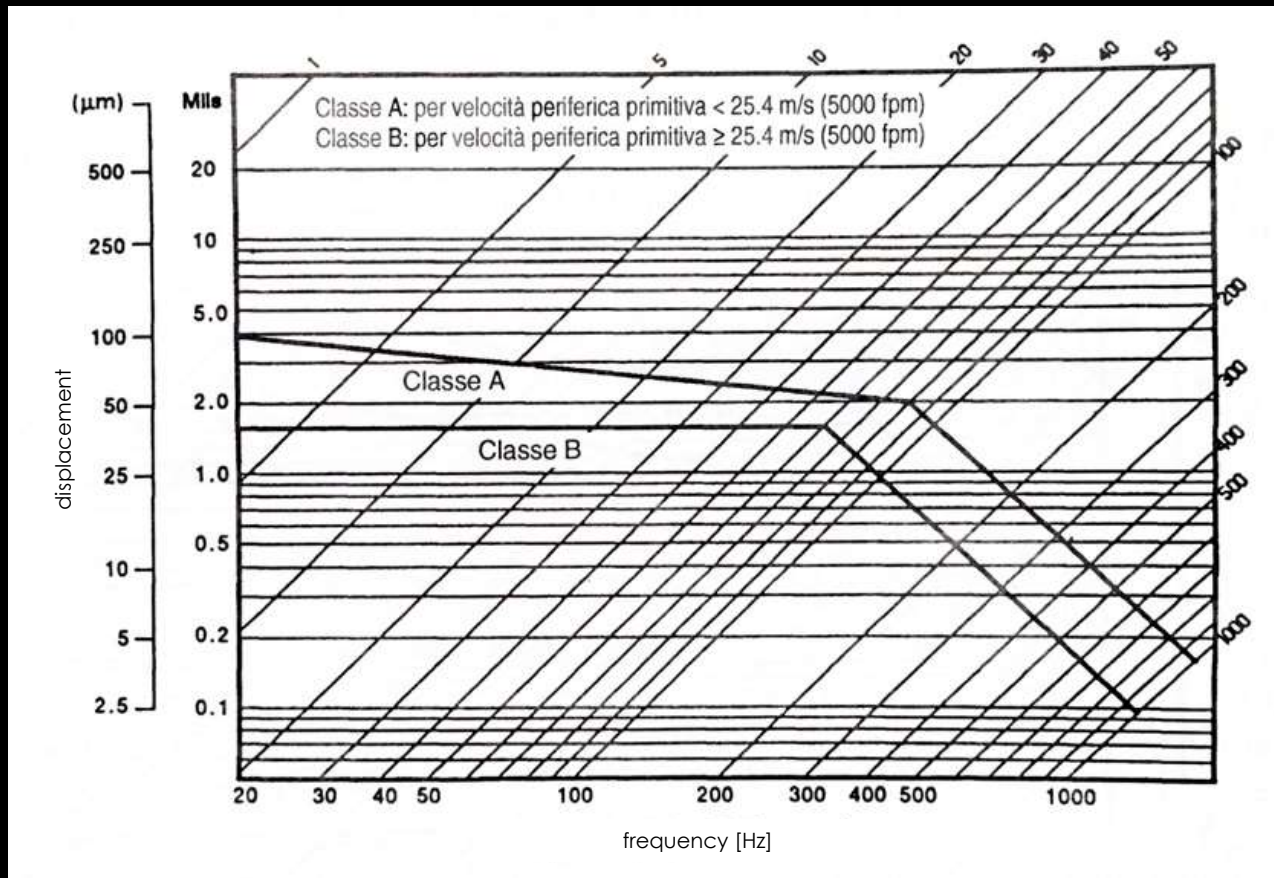
Example (Dudley)

A lightweight planetary gearbox of 10,000 Hz and 1000kW can see 40g and last 30,000 hours (note: 10,000Hz for 33 teeth -> 18,000rpm)



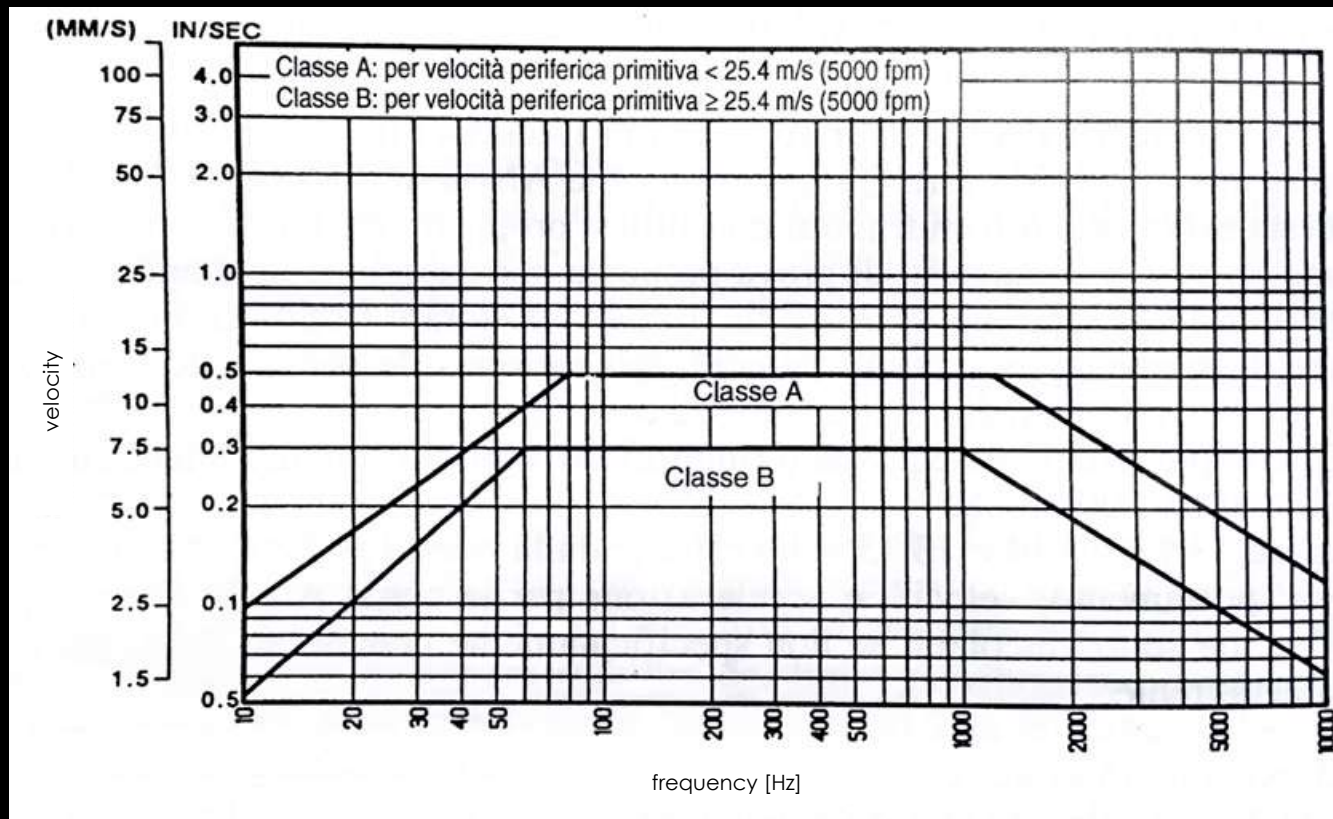
VIBRATIONAL LIMITS

Displacement amplitude normogram (AGMA 6000-A88)



VIBRATIONAL LIMITS

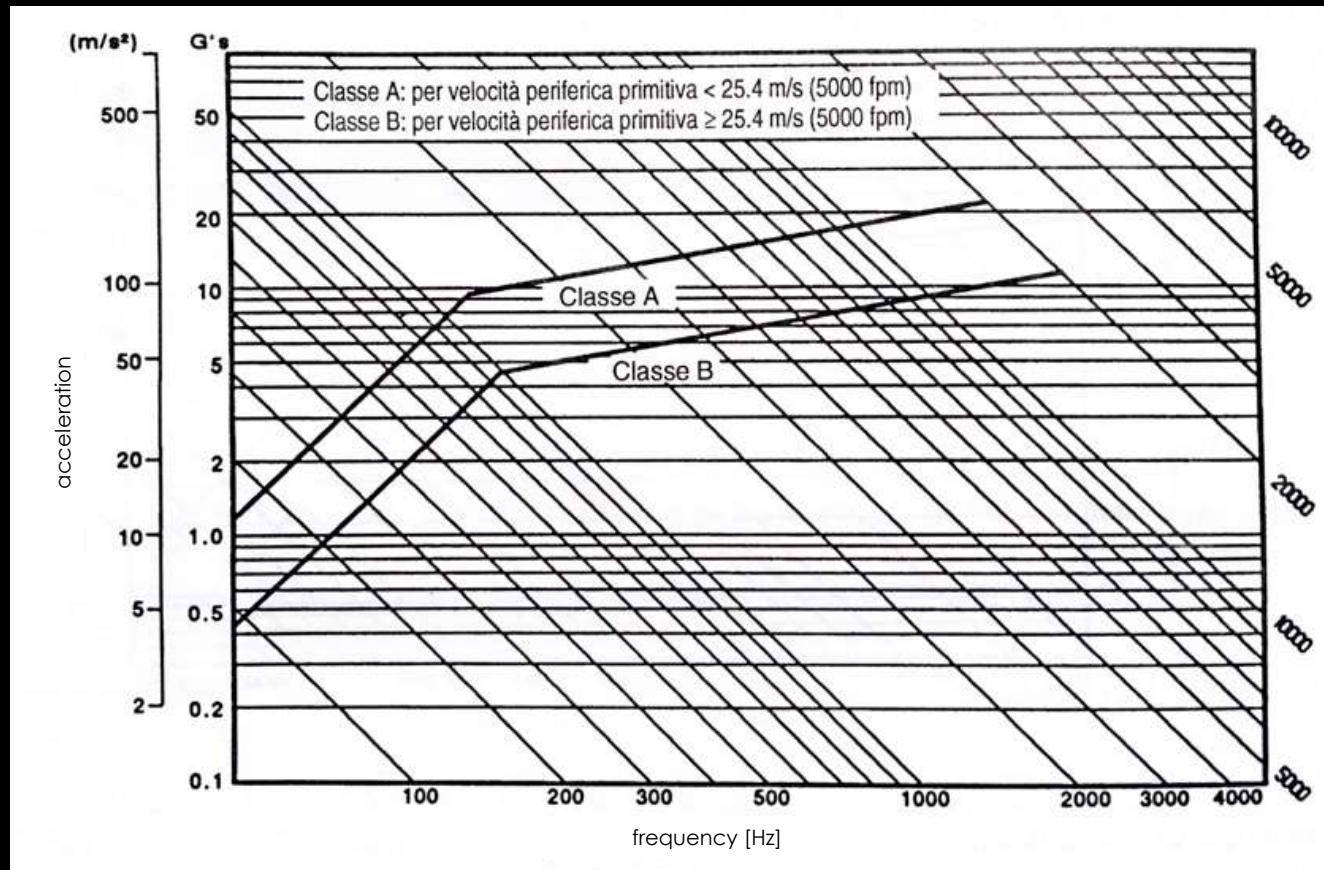
Normogram speed amplitudes (AGMA 6000-A88)



fpm: feeth per minute

## VIBRATIONAL LIMITS

Normogram amplitudes in acceleration (AGMA 6000-A88)



# NVH ISSUES

## QUALITY CLASSES: COMPARISON OF STANDARDS

more accurate

less accurate

STANDARD	CLASS												
	15	14	13	12	11	10	9	8	7	6	5	4	3
AGMA 390.03 (USA)	15	14	13	12	11	10	9	8	7	6	5	4	3
DIN 3962 (GERMANIA)	1	2	3	4	5	6	7	8	9	10	11	12	
<b>ISO 1328 (ITALIA)</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	
BS 4 (REGNO UNITO)					A <sup>1</sup>	A <sup>2</sup>	B	C	D				
3SEIS (FRANCIA)				A	B	C	D	E					
JIS B 1702 (GIAPPONE)				0	1	2	3	4	5	6	7	8	
KS B 1405 (COREA)				0	1	2	3	4	5	6	7	8	



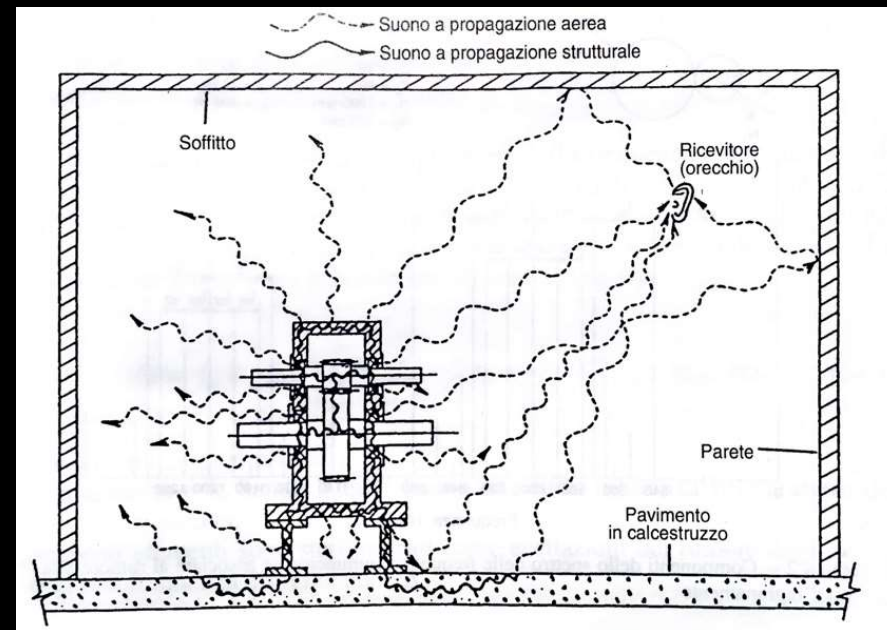
## Quality class: standards

- ISO 1328-1
  - Pitch
    - Single Pitch Deviation  $f_{pt}$
    - Cumulative Pitch Deviation  $f_{pk}$
    - Total Cumulative Pitch Deviation  $f_p$
  - Profile error
    - Total Profile Deviation  $f_a$
    - Profile Form Deviation  $f_{fa}$
    - Profile Slope Deviation  $f_{ha}$
  - Helix
    - Total helix deviation  $f_b$
    - Helix Form Deviation  $f_{fb}$
    - Helix Slope Deviation  $f_{hb}$
- ISO 1328-2
  - Total Radial Composite Deviation ( $F_i$ )
  - Runout Error of Gear Teeth ( $F_r$ )

## VIBRATION AND NOISE

### Paths

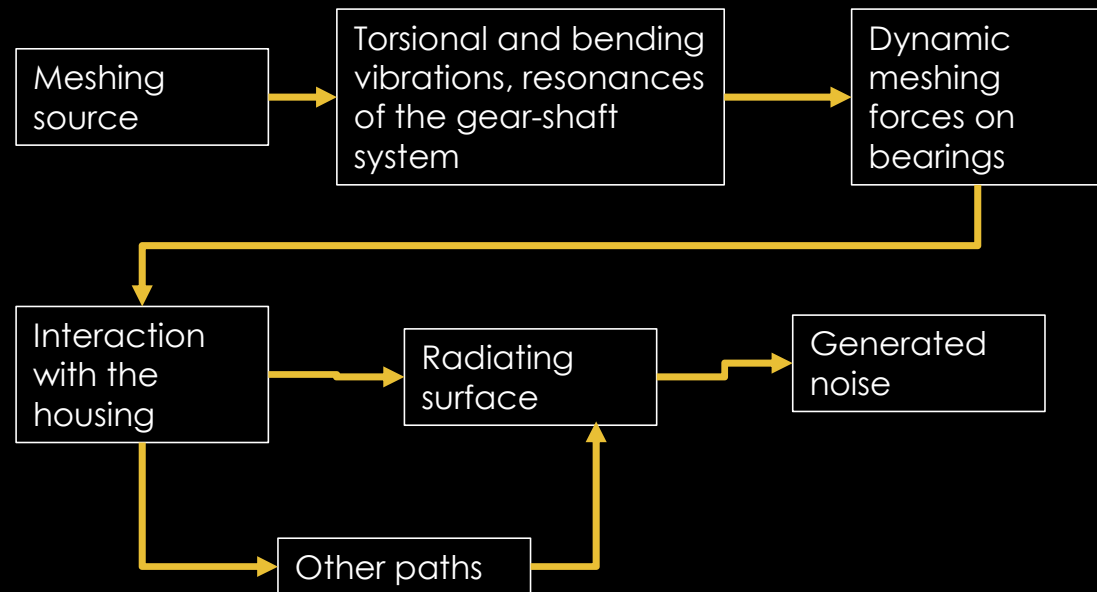
- The vibration source is produced by the meshing
- The vibration is transferred through forces on the shafts
- Then it is transferred to the housing
- Finally, the housing radiates sound into the environment



## VIBRATION AND NOISE

### Paths

- The vibration source is produced by the meshing
- The vibration is transferred through forces on the shafts
- Then it is transferred to the housing
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## VIBRATION AND NOISE

## NVH ISSUES

Suggestions to reduce emitted noise (Houser)

- Reduce gear excitation
- Imperfect action between conjugated surfaces causes vibration: OPTIMIZE MICRO-GEOMETRY
- Reduce vibration transmission paths between gear and casing
- Reduce the acoustic emissivity of the housing
- Change the environment where the gearbox operates: isolation and confinement devices

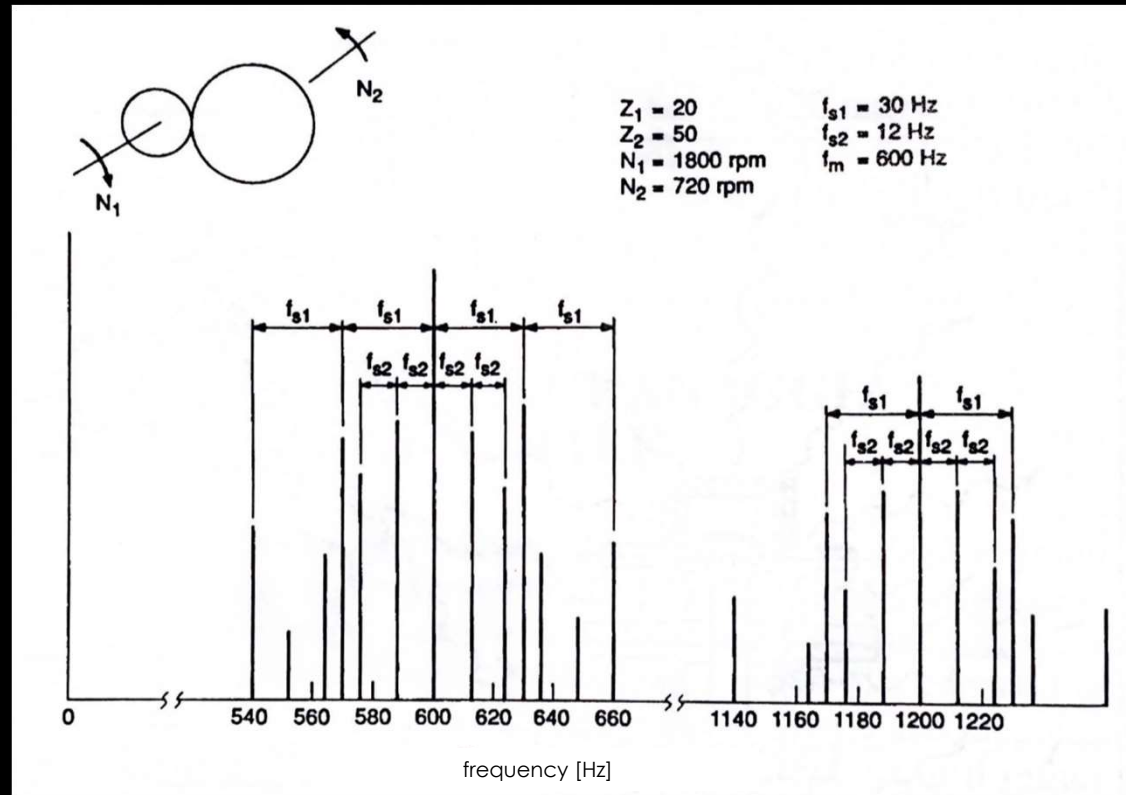


## VIBRATION AND NOISE

Most of the noise is related to:

## NVH ISSUES

- Mesh frequency
- Multiples of the gear frequency
- Side-bands (quasiperiodicity or amplitude modulation)



## VIBRATION AND NOISE

Most of the noise is related to:

Shaft frequency 1

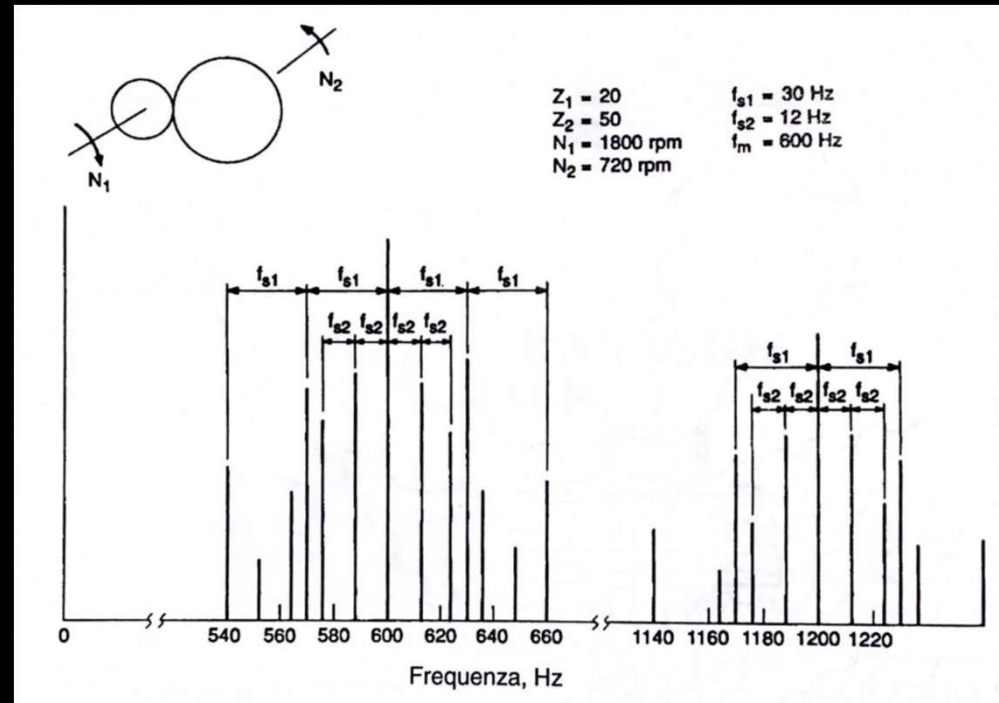
$$f_{s1} = \frac{N_1}{60}$$

Shaft frequency 2

$$f_{s2} = \frac{N_2}{60} = \frac{f_{s1} z_1}{z_2}$$

Mesh frequency

$$f_m = f_{s1} z_1 = f_{s2} z_2$$



$N_i$  Shaft rotation speed  $i$

$z_i$  number of teeth wheel  $i$

## NVH ISSUES

## VIBRATION AND NOISE

## NVH ISSUES

- Peaks in the spectrum (harmonics) are determined at integer multiples of the meshing frequency  $f_m$ ,  $2f_m$ ,  $3f_m$ ,  $4f_m$
- Sidebands generally depend on the rotation frequency of the wheel:

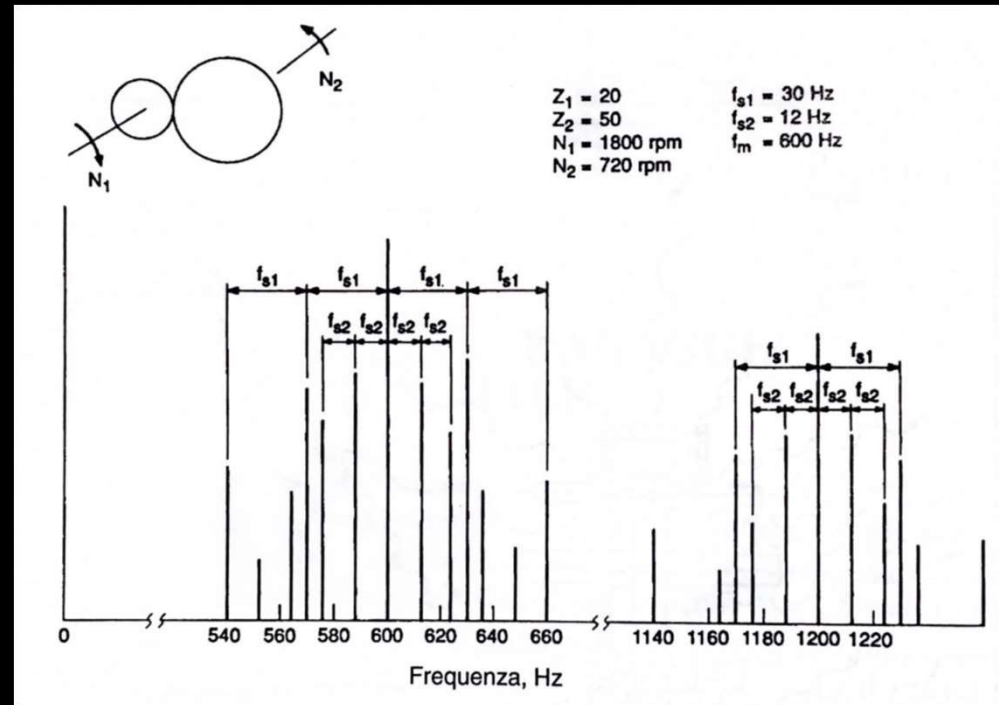
$$f_m \pm n f_{s1}$$

$$f_m \pm n f_{s2}$$

$$2f_m \pm n f_{s1}$$

$$2f_m \pm n f_{s2}$$

And so for every harmonics

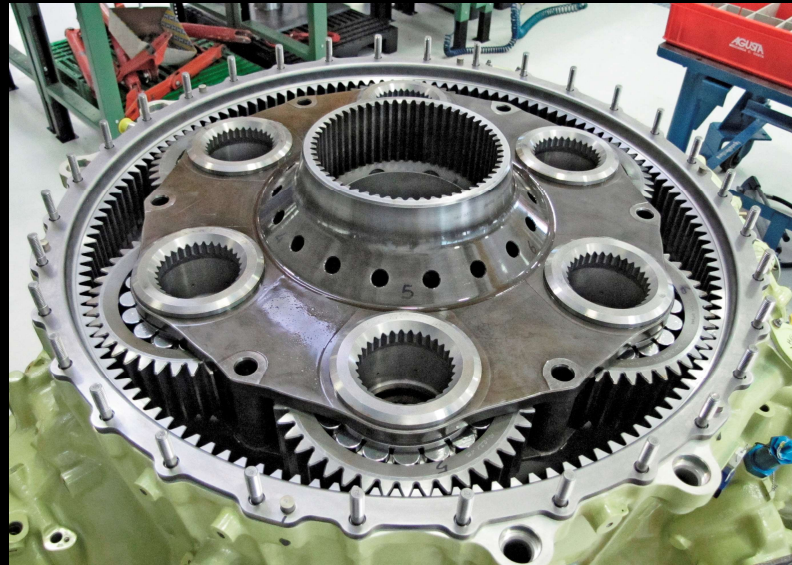




## VIBRATION AND NOISE

## NVH ISSUES

Mesh frequency for planetary



$$\omega_{\text{mesh}} = \omega_{\text{sun}} \frac{z_{\text{sun}} z_{\text{ring}}}{(z_{\text{sun}} + z_{\text{ring}})}$$

### Elements that affect vibration at the mesh frequency

- Variation of mesh stiffness
  - Transmission error
- Shocks at the beginning of contact
- Dynamic mesh forces
- Effects of frictional forces
- Lubricant entrapment

## VIBRATION AND NOISE

## NVH ISSUES

### Transmission error

The literature indicates the **TRANSMISSION ERROR TE** as the single **most important factor** in the generation of noise in gears

Difference between the actual position of the driven wheel and the position it would occupy if the two wheels were perfectly conjugated (perfect and rigid wheels)

Angular transmission error  $TE = \theta_2 - \frac{z_2}{z_1} \theta_1$  rad

Transmission error on the action line  $TE = R_{b1} \left( \theta_2 - \frac{z_2}{z_1} \theta_1 \right)$  mm o in.



## VIBRATION AND NOISE

## NVH ISSUES

### Transmission error

It depends on several factors:

If the teeth are unloaded **TE** depends on the

- inaccuracies:
- Profile errors
- Pitch errors
- eccentricity

If the teeth are loaded **TE** still depends on the inaccuracies and errors as before, but not only:

- 
- It depends a lot on the stiffness of the meshing
- the elasticity-dependent part of the TE can be mitigated with appropriate reliefs

## VIBRATION AND NOISE

## NVH ISSUES

### Transmission error under load

Under operating conditions the two effects contributing to TE are:

- constant component due to the average elasticity of the tooth
  - affects the definition of the reliefs
  - not very significant as direct impact on noise
- Variable component that depends on geometry and mesh stiffness
  - Significant impact on noise produced

## VIBRATION AND NOISE

## NVH ISSUES

### Transmission error under LTE load

- The mesh stiffness is proportional to the average number of teeth in contact (contact ratio)
- The **contact ratio** therefore plays an important role
- It must be taken into account that under load the contact ratio changes (is reduced) compared to the geometric / kinematic one
- Theoretically, the contact ratio would seem to have to be an integer number (if we assume a piecewise constant stiffness)
- Wheels with exactly integer contact ratio are rarely designed

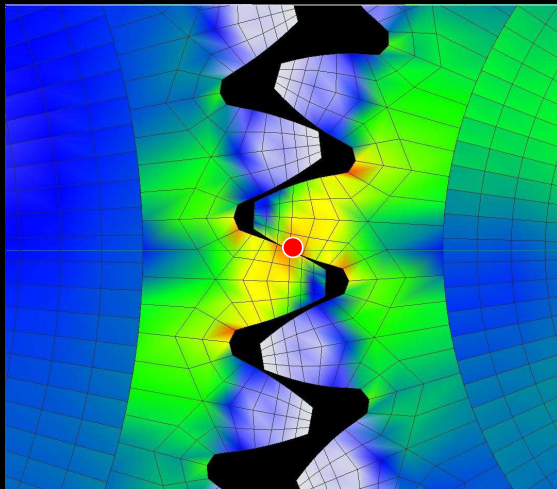


# NVH Issues

## LTE: ELASTICITY EFFECT

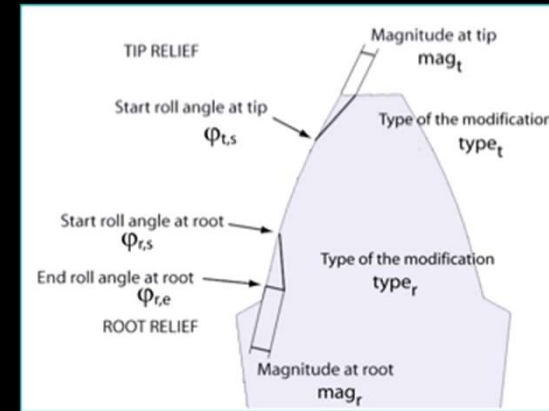
- Errors and profile modifications
- Elastic deformations

$$1 < \varepsilon_{\alpha} < 2$$

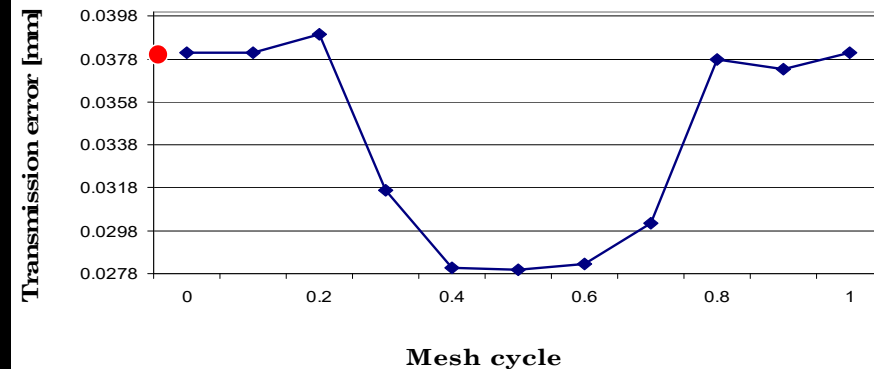


The static transmission error is periodic in the gear cycle unless there are constructive errors

WE TRY TO REABSORB IT WITH THE RELIEFS



## STATIC TRANSMISSION ERROR



**BEFORE PROCEEDING IT IS USEFUL TO  
RECALL CONCEPTS OF WHEEL GEOMETRY**

**MICRO-GEOMETRIC MODIFICATIONS**



# MICRO GEOMETRIC MODIFICATIONS

F. Pellicano – A- Zippo - [francesco.pellicano@unimore.it](mailto:francesco.pellicano@unimore.it) - [antonio.zippo@unimore.it](mailto:antonio.zippo@unimore.it) +39 3316074466



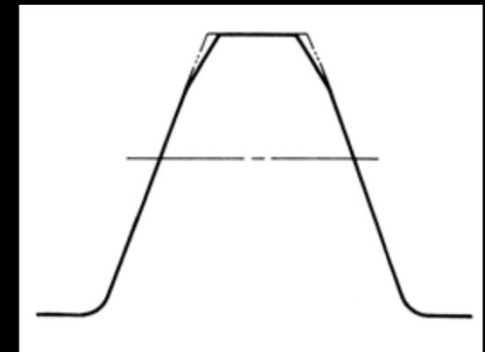
# PROFILES MODIFICATIONS

There are many unique technical words related to gearing. Also, there are various unique ways of modifying gears. This section introduces some of most common methods. (KHK Reference Handbook)

# PROFILE MODIFICATIONS

## Tooth Profile Modification

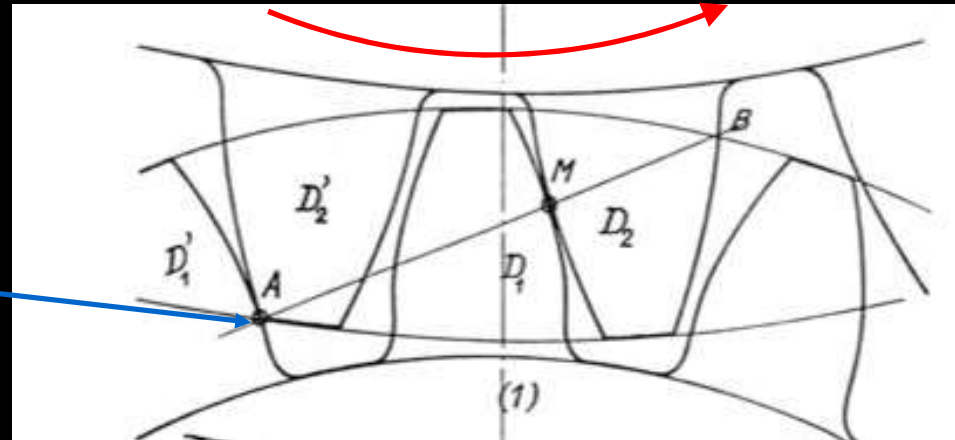
- Tooth Profile Modification generally means adjusting the addendum.
- Tooth profile adjustment is done by chamfering the tooth surface in order to make the incorrect involute profile on purpose.
- This adjustment, enables the tooth to vault when it gets the load, so it can avoid interfering with the mating gear.
- This is effective for reducing noise and longer surface life.
- However, too much adjustment may create bad tooth contact as it functions the same as a large tooth profile error.



# MESHING ISSUES

Error-free profile

Start regular contact



This is true if:

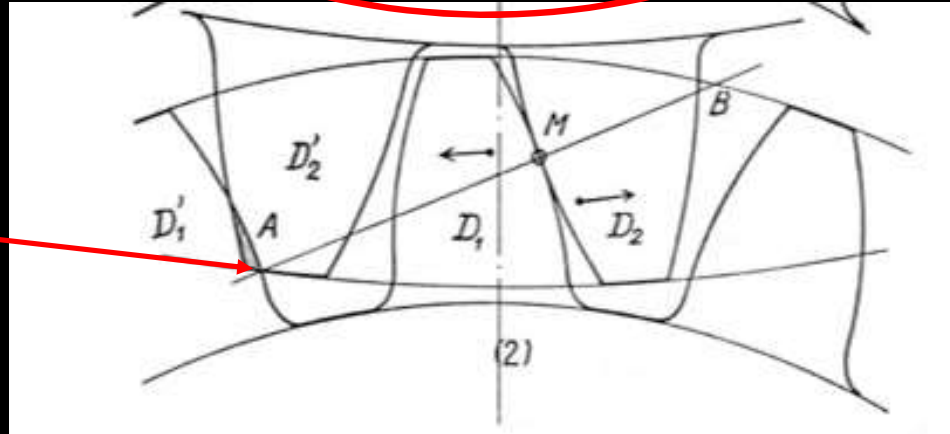
- There are no constructive errors
  - errors in pitch, profile, assembly, alignment
- Elastic deformations are sufficiently small
  - Low loads compared to sustainable maxima



# MESHING ISSUES

Teeth with error or deformation

Approach:  
irregular contact



- At the approach  $D'_1$  and  $D'_2$  the two profiles are not conjugated for some reason
  - Pitch error, profile error, deformation
- Impact takes place at the contact start
- The tooth end of wheel 2 tends to penetrate the tooth side of wheel 1
- Pressure peaks are found locally
- Problems of poor lubrication, wear, early pitting may arise locally
- Considerable noise is produced

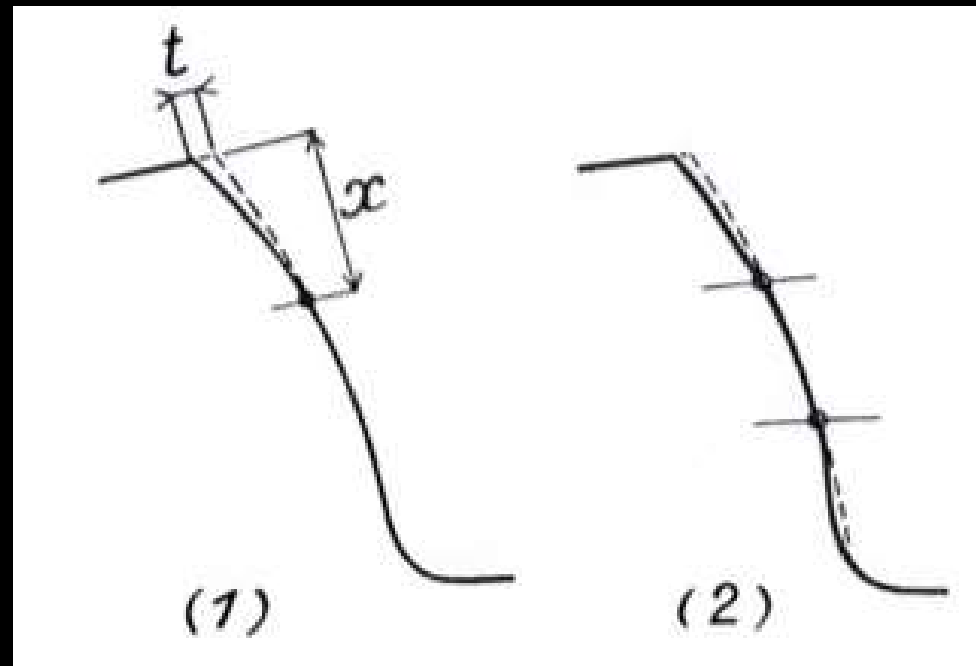
# MESHING ISSUES: MITIGATION TECHNIQUES

## TIP AND ROOT RELIEF

- Henriot suggests a slight tip relief on the driven wheel or alternatively
- a slight root relief on the driver wheel to avoid incoming impacts

In addition

- He also suggests a slight tip relief on the driver wheel to reduce the pressures during the exit



Tip relief

Tip and root relief

# MESHING ISSUES: MITIGATION TECHNIQUES

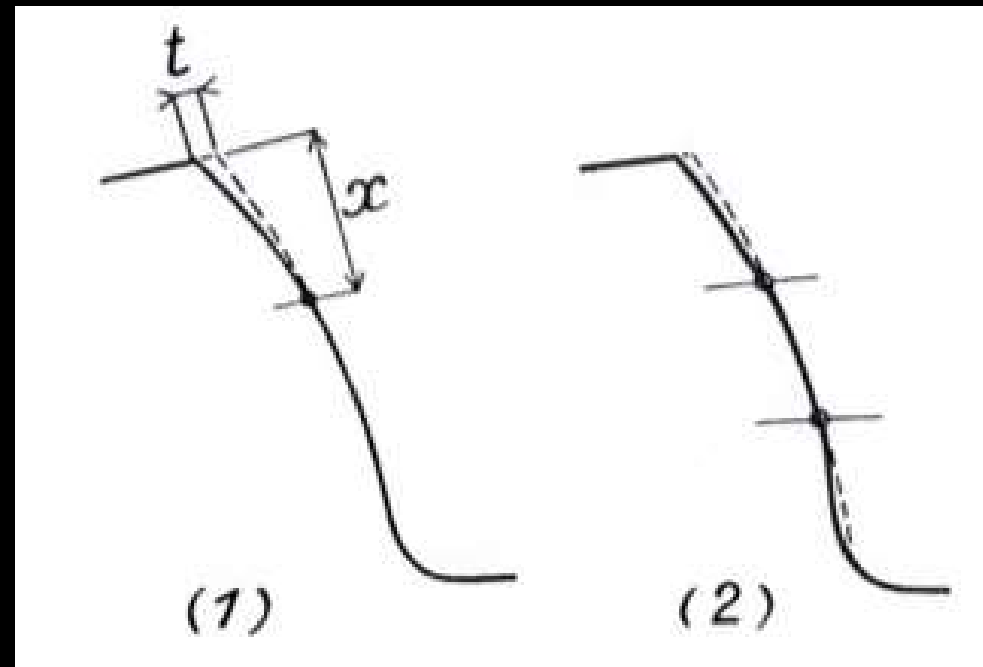
## TIP AND ROOT RELIEF

At high load and high speed it is good to have reliefs on both wheels, with priority on the driven

Henriot gives a qualitative indication of the relief magnitude

$1 \mu\text{m}$  for  $20 \frac{\text{N}}{\text{mm}}$  unitary load

Suggestion for the relief  $t$



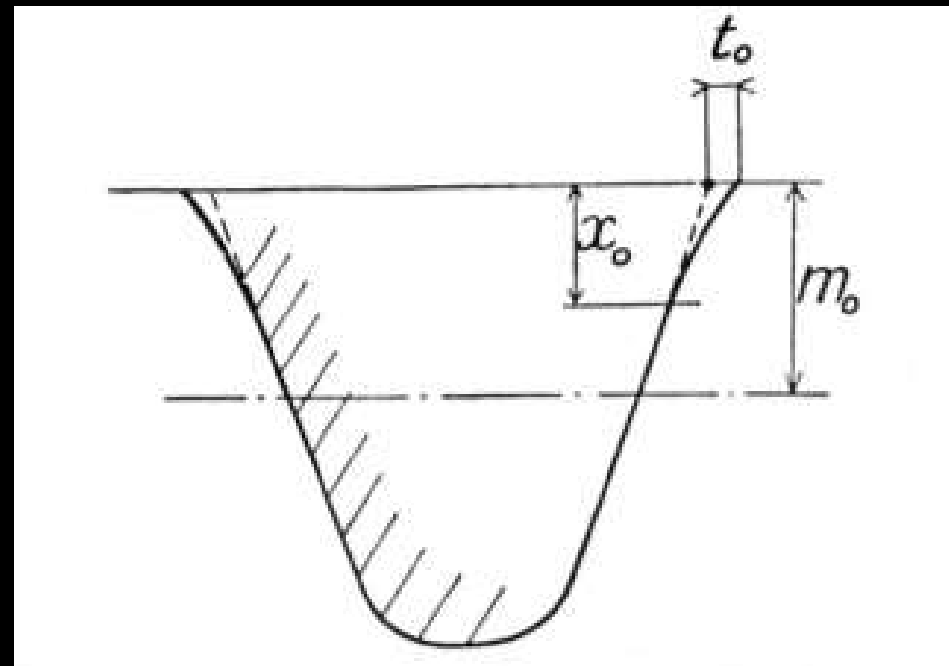
Tip relief

Tip and root relief

# PROFILE MODIFICATIONS

## TIP AND ROOT RILIEF

Profile modifications are usually obtained by grinding  
However, they can also be obtained with modified rack

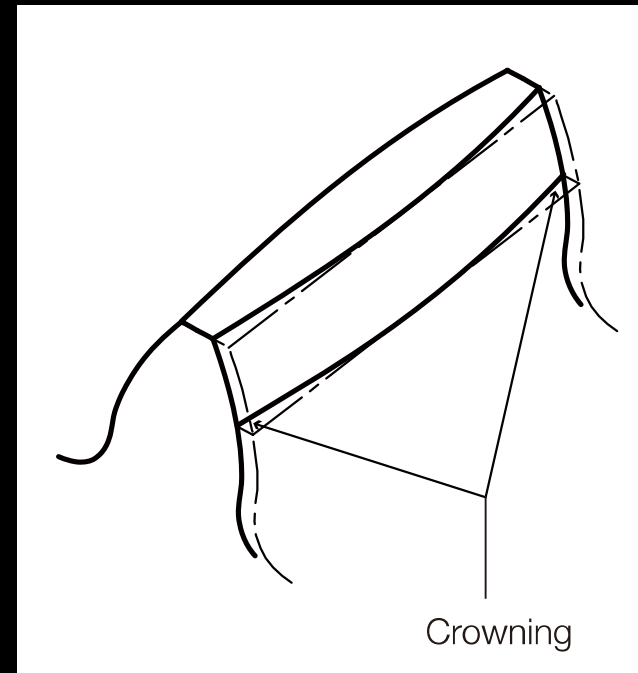




# TOOTH MODIFICATIONS

## Crowning and End Relief

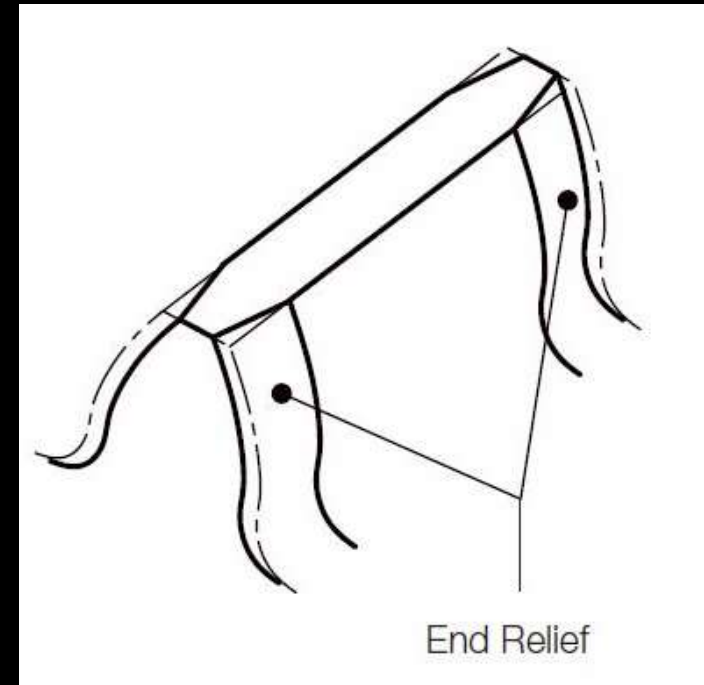
- Crowning is the removal of a slight amount of the tooth from the center on out to the reach edge, making the tooth surface slightly convex.
- This method allows the gear to maintain contact in the central region of the tooth and permits avoidance of edge contact.
- Crowning should not be larger than necessary as it will reduce the tooth contact area, thus weakening the gears strength.
- End relief is the chamfering of both ends of tooth surface.



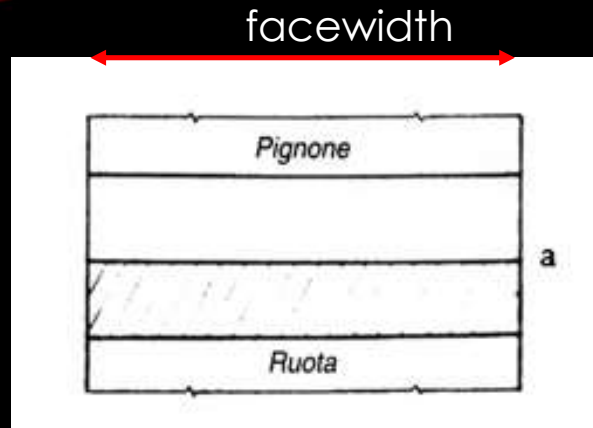
# TOOTH MODIFICATIONS

## Crowning and End Relief

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# CONTACT CONDITIONS ALONG THE WIDTH



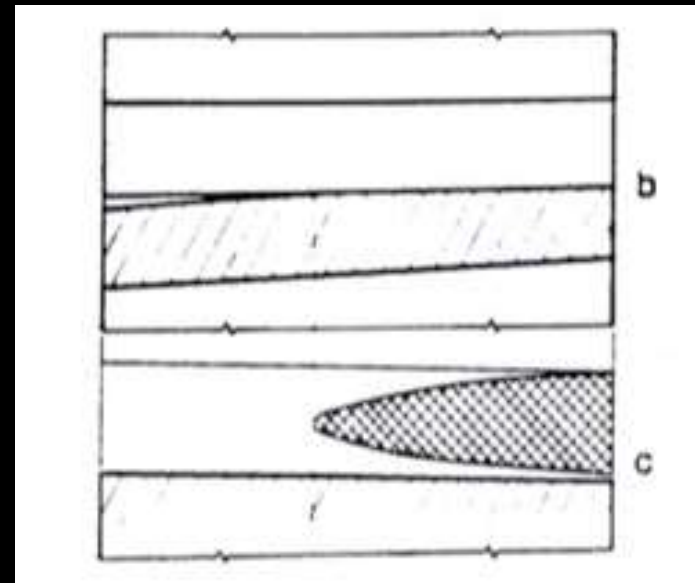
Regular meshing  
Perfectly parallel wheels

**Wheels with misalignment or shaft failure**

**Remedies**

- Symmetrical crowning for weak distortions
- helix correction for large distortions

Crowning increases Hertzian pressure



# TOOTH MODIFICATIONS

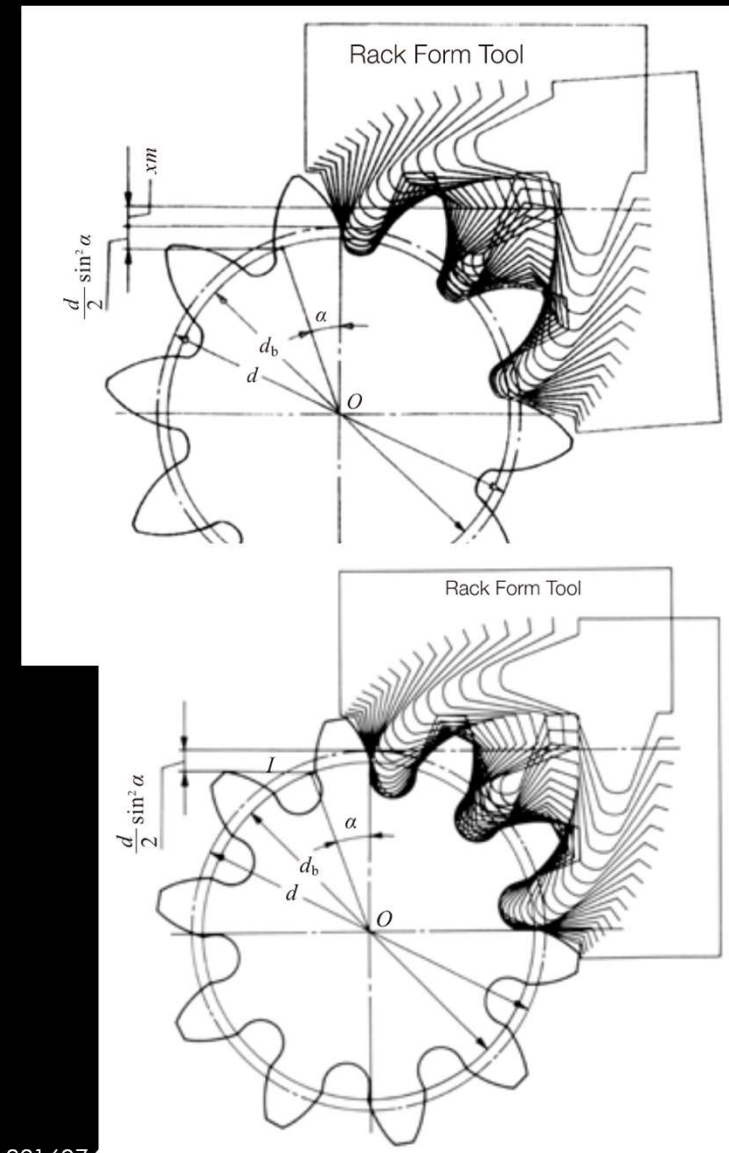
## Topping and Semitopping

In topping, often referred to as top hobbing, **the top or tip diameter of the gear is cut simultaneously with the generation of the teeth.**

The figures indicate topping and generating of the gear by rack type cutters.

An advantage is that there will be no burrs on the tooth top.

Also, the tip diameter is highly concentric with the pitch circle.



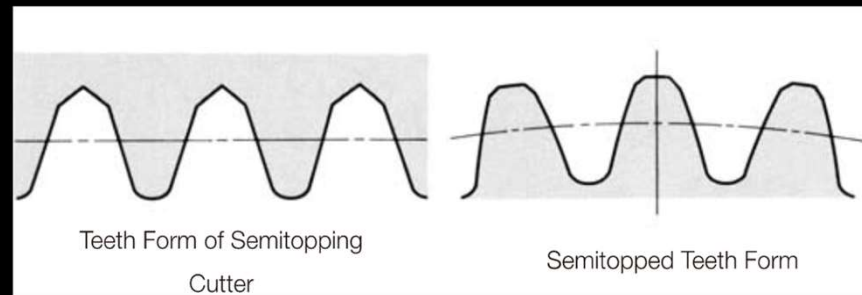


# PROFILE MODIFICATIONS

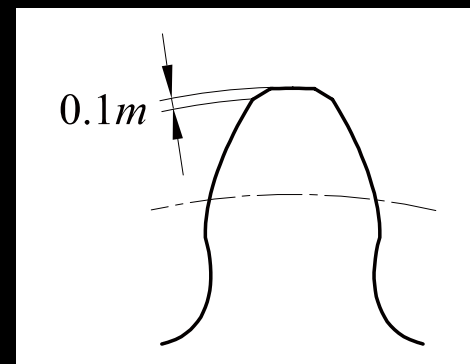
## Topping and Semitopping

**Semitopping** is the chamfering of the tooth's top corner, which is accomplished simultaneously with tooth generation.

The Figs. show a semitopping cutter and the resultant generated semitopped gear. Such a tooth end prevents corner damage and has no burrs (bave).



Semitopping Cutter and the Gear Profile



Magnitude of Semitopping



**NVH ISSUES**

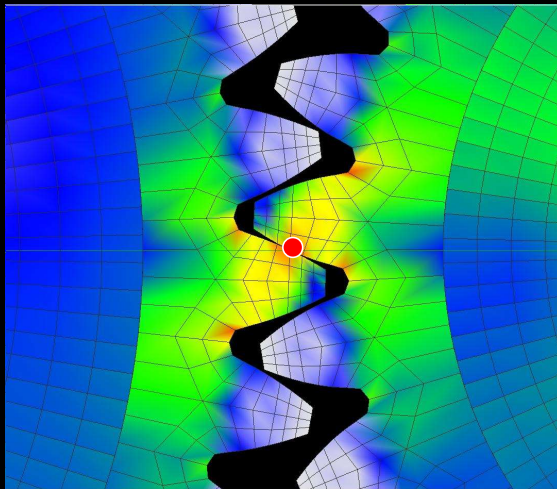
**LOADED TOOTH CONTACT  
ANALYSIS  
LTCA**

# NVH Issues

## LTE: ELASTICITY EFFECT

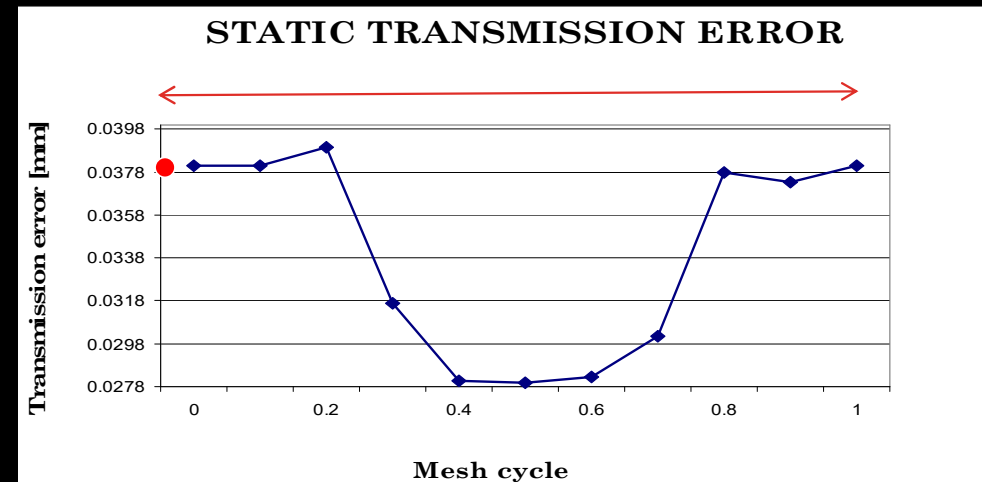
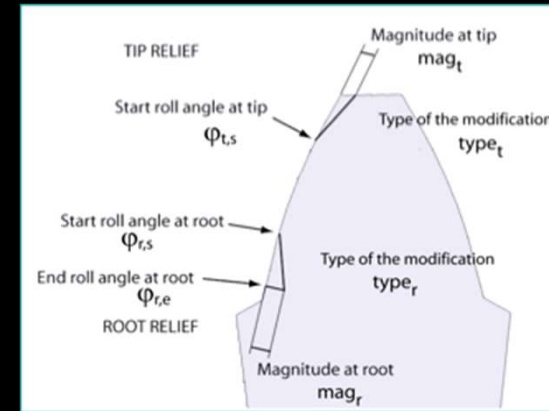
- Errors and profile modifications
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- 

$$1 < \varepsilon_{\alpha} < 2$$



The static transmission error is periodic in the gear cycle unless there are constructive errors

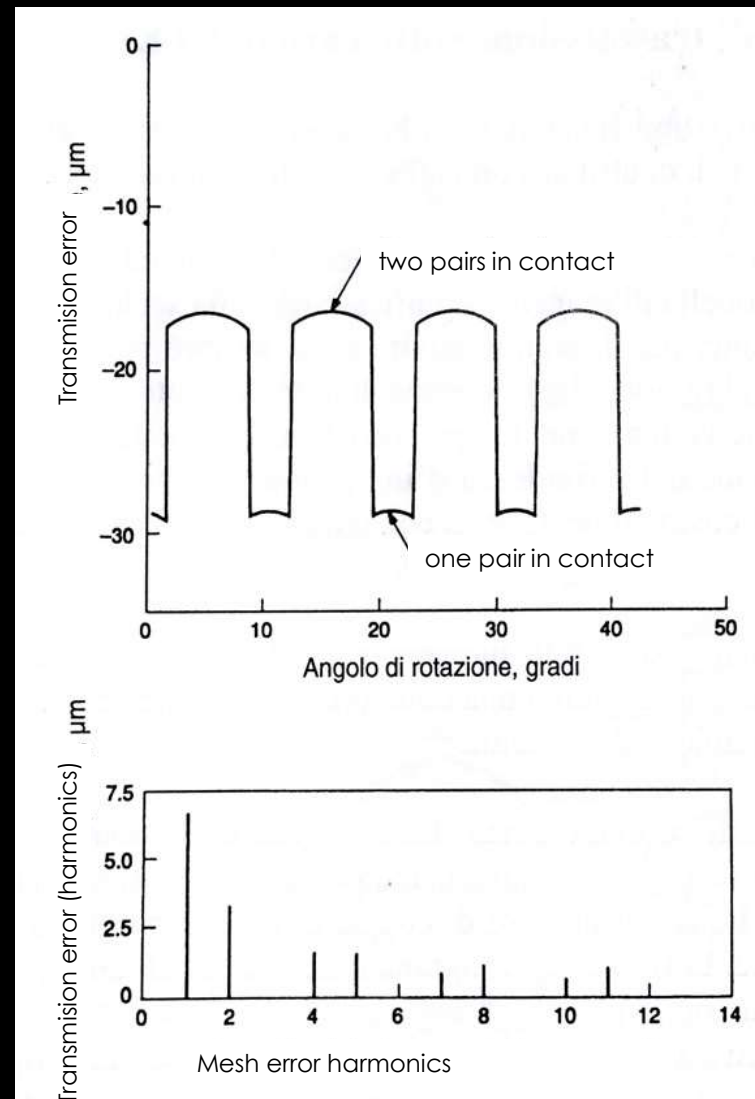
WE TRY TO REABSORB IT WITH THE RELIEFS



# Problematiche NVH

## LTE Transmission Error Under Load

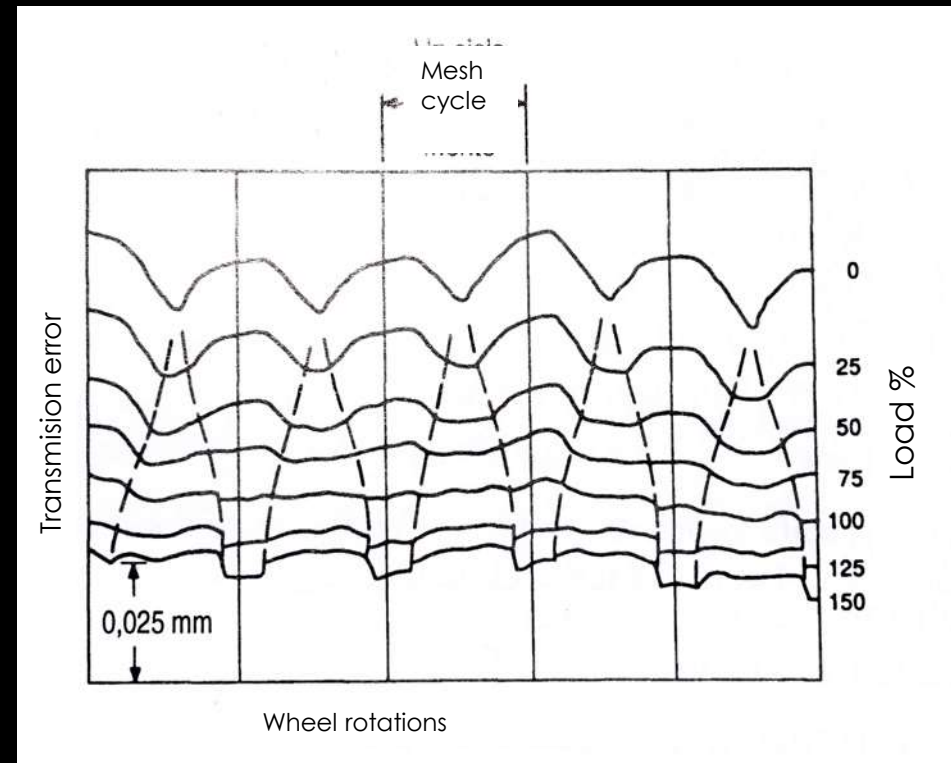
- We often assume piecewise constant stiffness (or almost)
- The transition between a pair of teeth in contact and two pairs in contact is modelled with a jump
- The harmonics of the transmission error are evaluated
  - The first harmonic usually dominates
  - With the reliefs it is possible to lower the first two-four harmonics





## LTE Transmission Error Under Load

- The reliefs modify the meshing
- Can be intrusive enough to change kinematics
- If they change the kinematics worsen the response to very low torque
- If they are well made at nominal load they reduce the TE on some aspect:
  - Peak peak
  - Single harmonics or average



## LTE Transmission Error Under Load

### Without reliefs

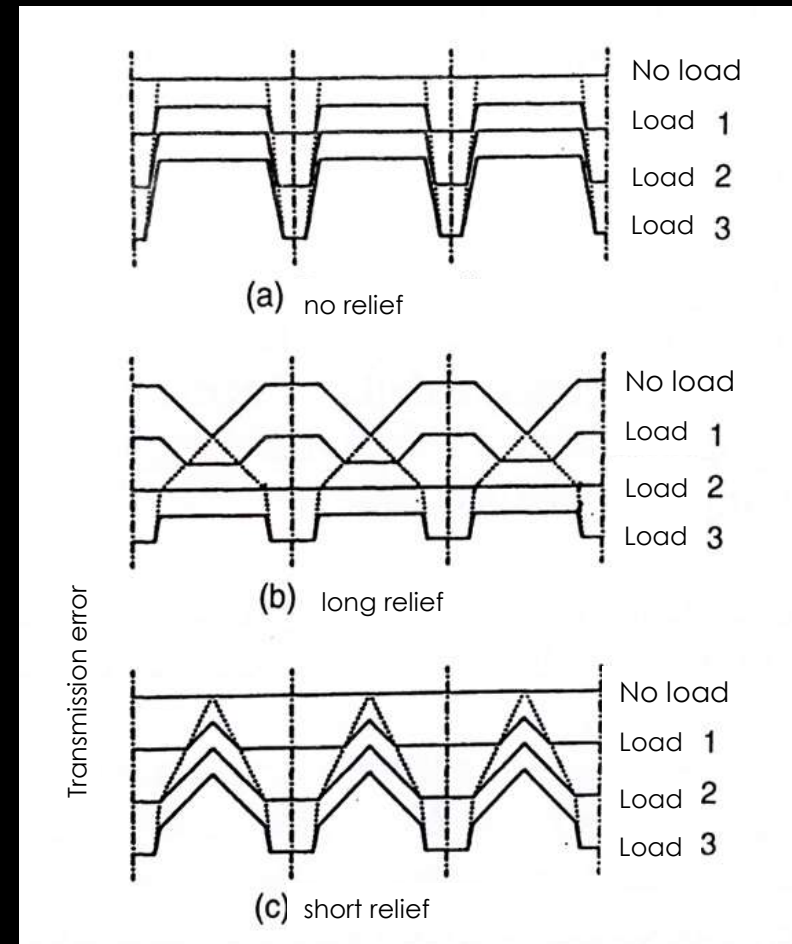
- Without load: perfect kinematic
- With load: LTE increasing

### Long relief

- No load (or low loads): kinematics not perfect, non-homokinetic transmission, possible noise problems
- With load: very low LTE compared to pure involute wheels

### Short relief

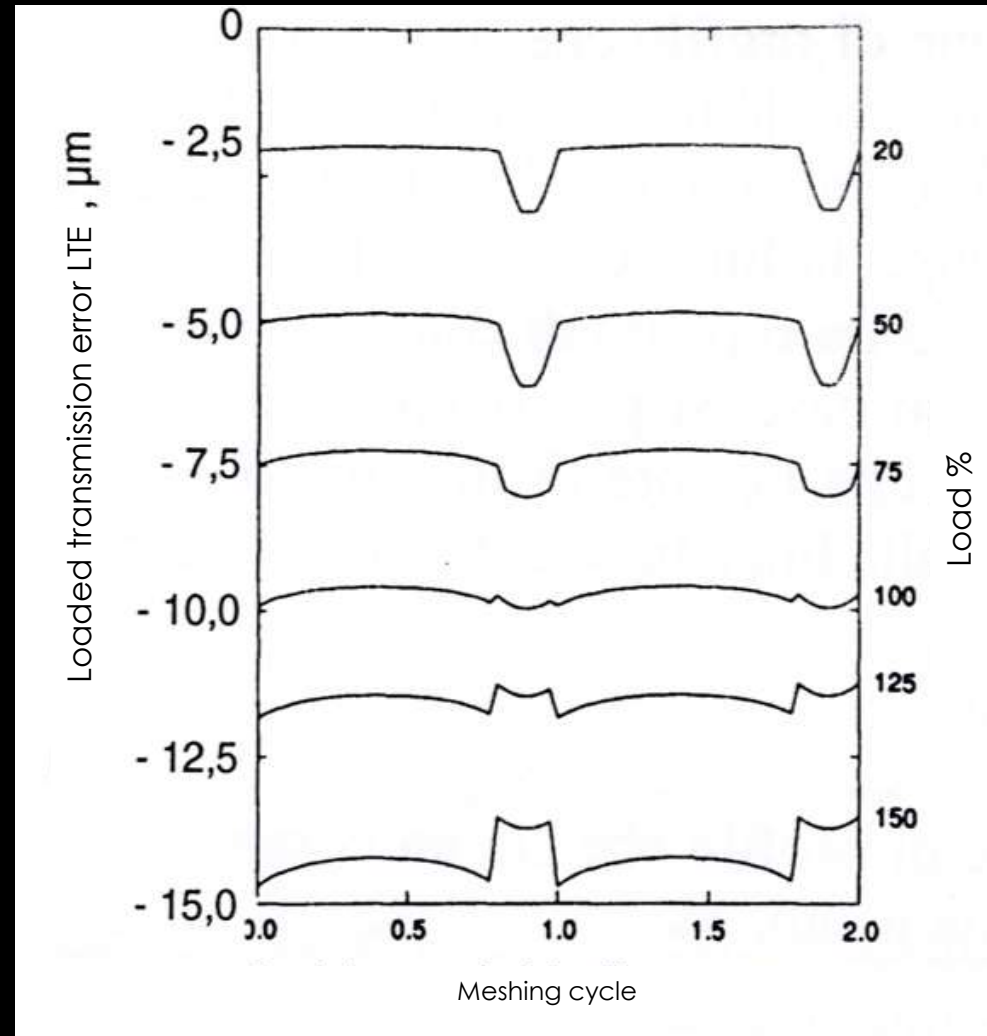
- Without load: perfect kinematic
- With load: LTE increasing
- **Much less effective than long modifications at nominal load**



## NVH Issues

### LTE Load Sensitivity

- Spur gears
- High Contact Ratio (2.2)
- In the figure simulated results (Houser non-FEM approach)
- The improvements are clear in all conditions compared to a gear with a lower gear ratio (e.g. 1.6-1.8)
- However, the constructive tolerances must be very tight



# HARRIS MAPS

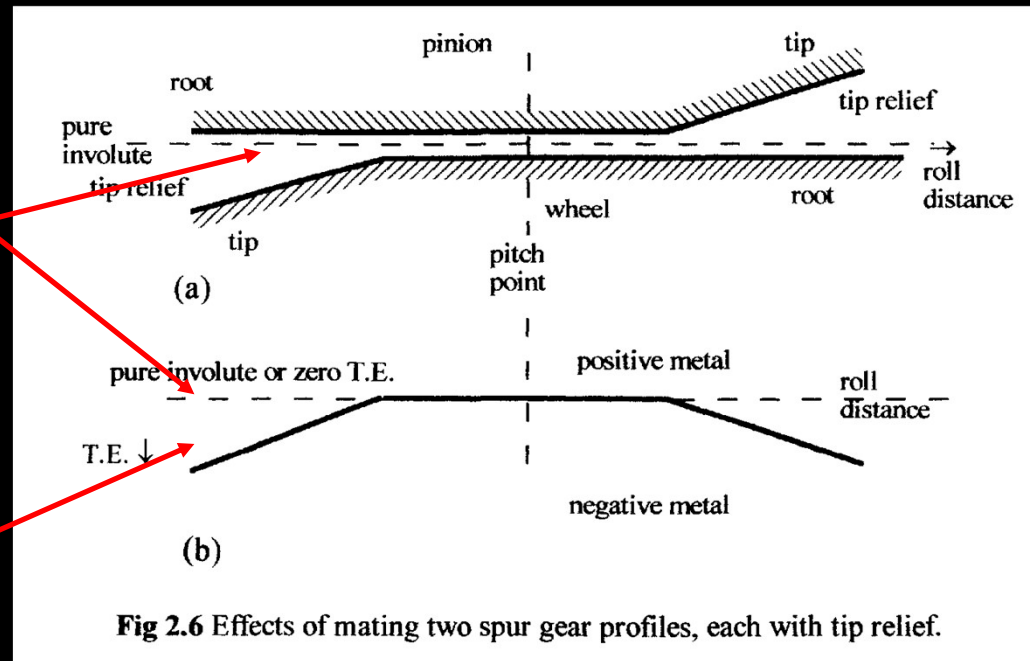
# NVH ISSUES

Effect of reliefs on transmission error

Unloaded wheels

Pure involute reference line

Deviation (TE) due to material removal (relief)



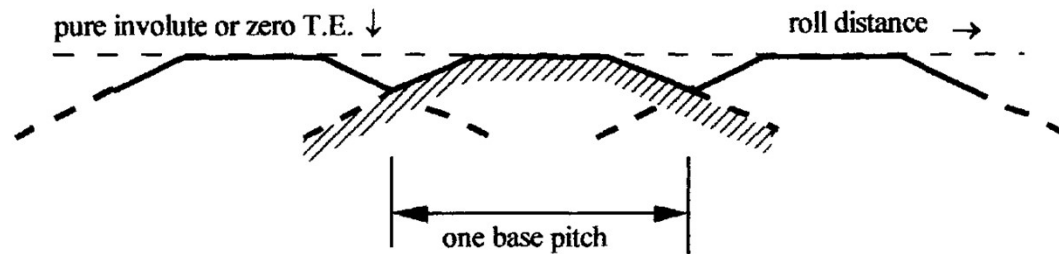


## HARRIS MAPS

## NVH ISSUES

Effect of reliefs on transmission error

Unloaded wheels



**Fig 2.7(a)** Effect on T.E. of handover to successive teeth when there are no elastic deflections.

Effect of meshing of successive teeth

It is also called "Intentional transmission error"

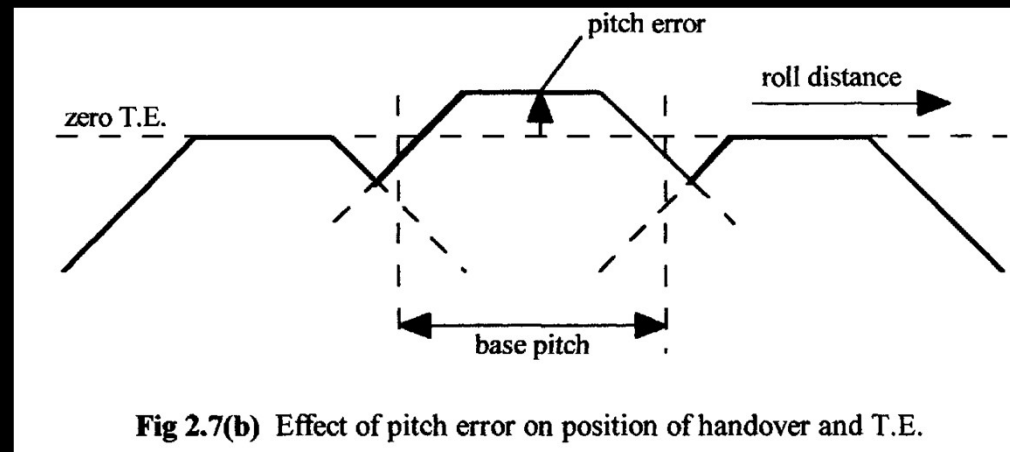
Note: In the experimental measurement, the dotted line is lost

## HARRIS MAPS

Effect of reliefs on transmission error

Unloaded wheels

## NVH ISSUES



Effect of meshing of successive teeth  
Combination with pitch error

## HARRIS MAPS

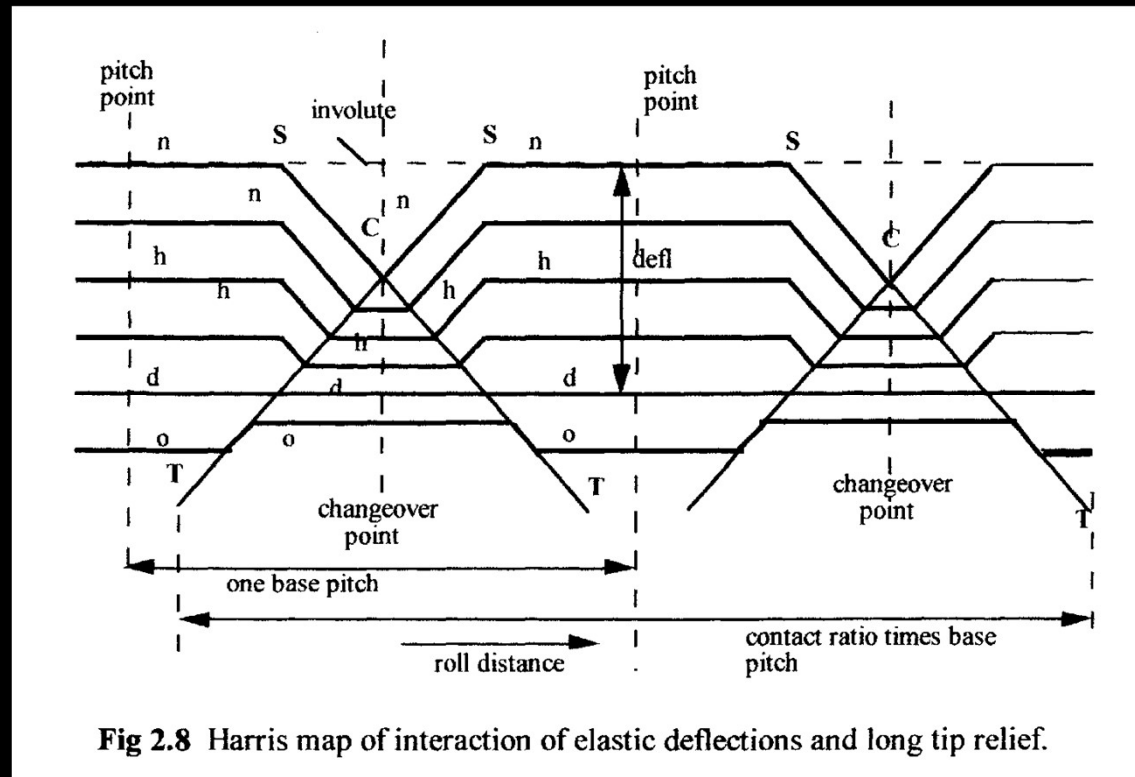
## NVH ISSUES

### Effect of reliefs on transmission error

#### Loaded wheels

One tries to predict the TE under load

It assumes constant stiffness!  
But the distribution of the load (two or more teeth) is taken into account.



Effect of reliefs on transmission error

Loaded wheels

- No load
- Half load
- Design load  
TE cancelled
- Overload  
TE positive  
when load is  
shared

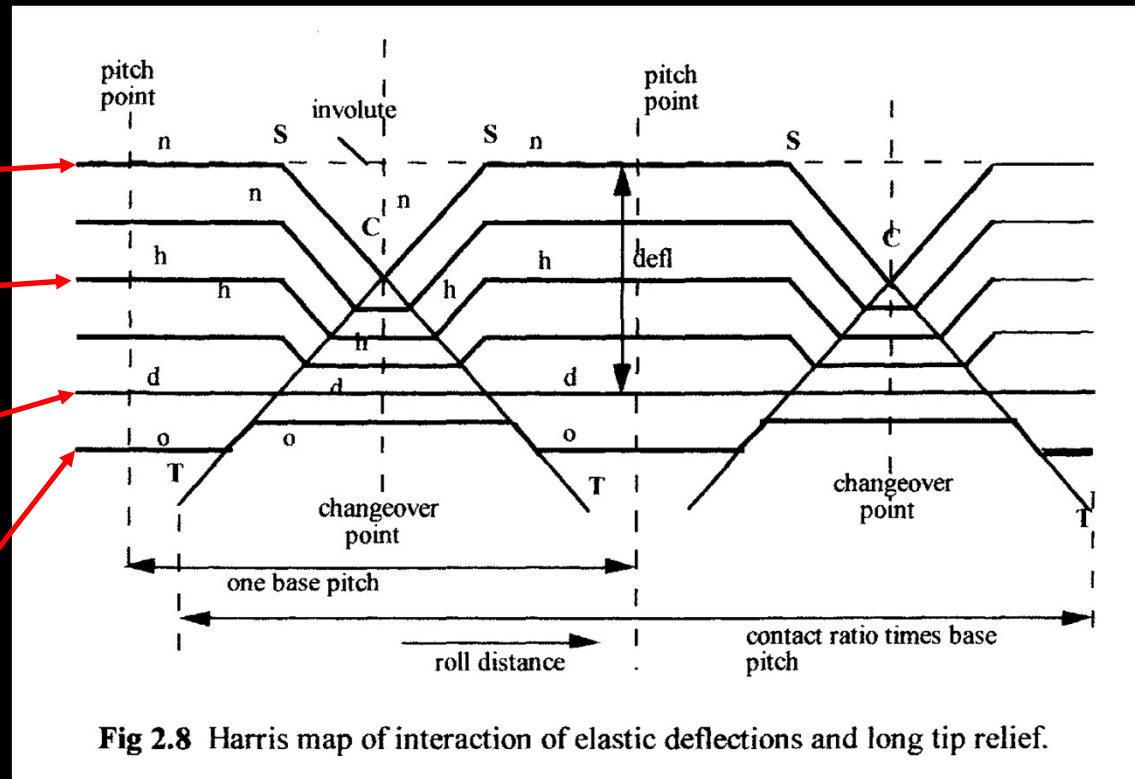


Fig 2.8 Harris map of interaction of elastic deflections and long tip relief.

long relief

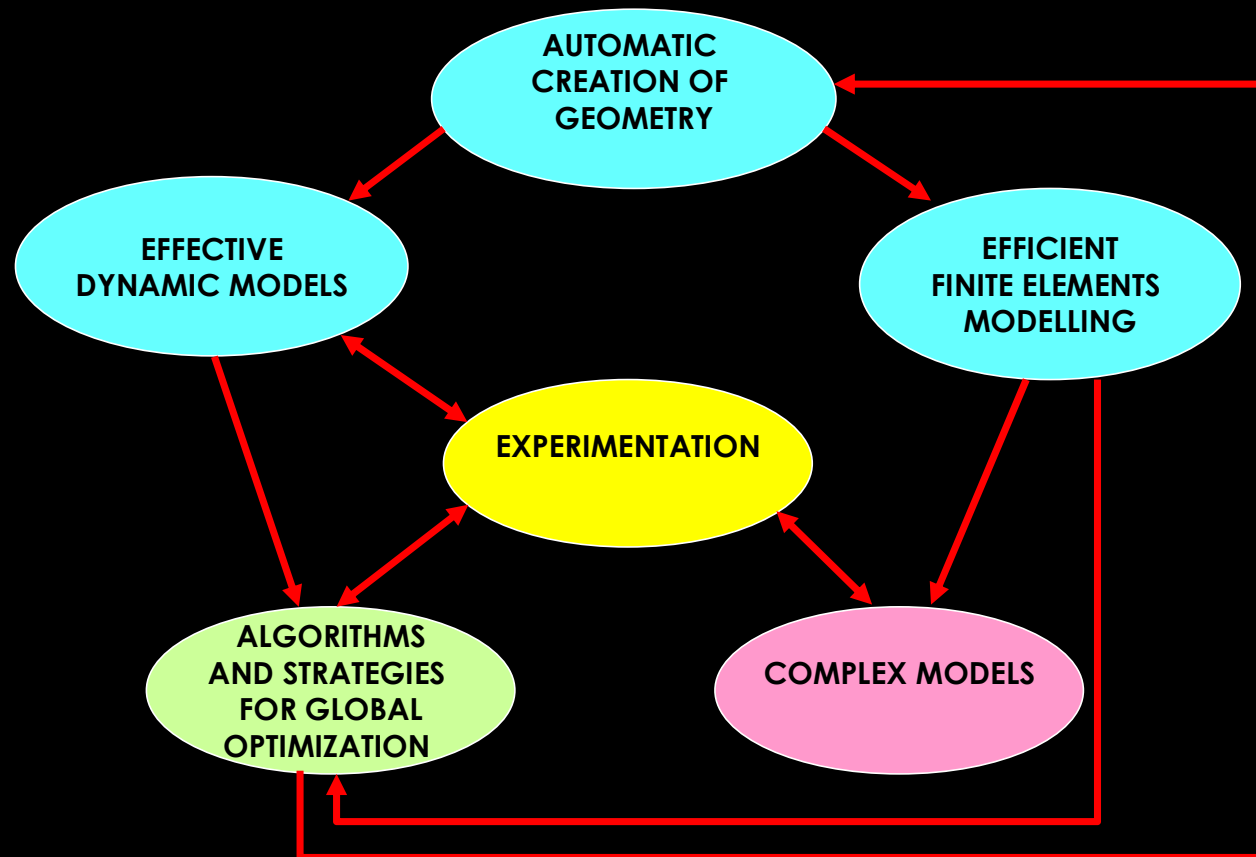




NVH ISSUES

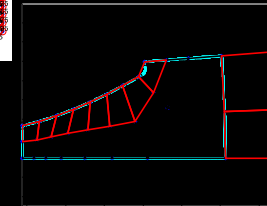
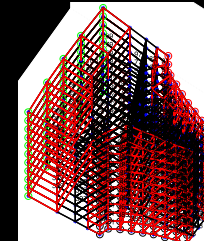
# MODERN APPROACH TO RELIEF OPTIMIZATION

# NVH ISSUES



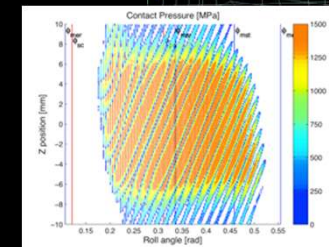
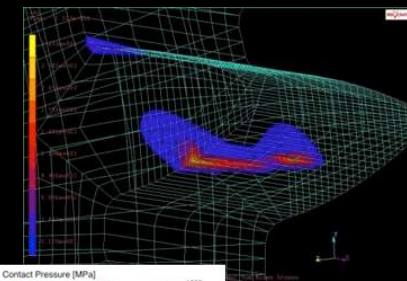
## Automatic Geometry Creation

- 3D geometries (envelope by points) NURBS methodology
- Arbitrary corrections (reliefs, crowning) misalignments



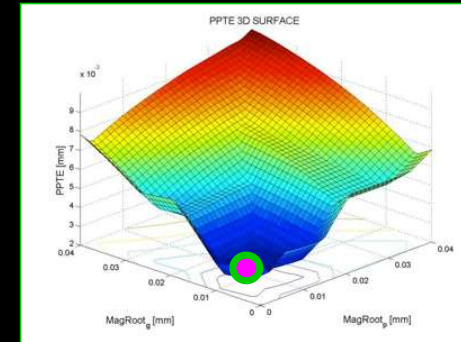
## Automatic creation of FEM models

- 2-3D ready-2-run models
- LTCA Load Contact Analysis
- Profile correction and misalignment effects
- Fatigue analysis



## Gear optimization

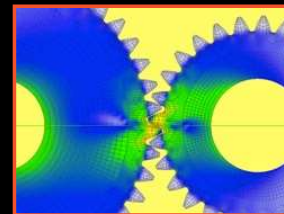
- Static and dynamic
- Genetic algorithms
- Random search+linear prog



## Dynamic models

- Natural frequencies and modes of vibration
- Nonlinear response

Gear trains



Planetary



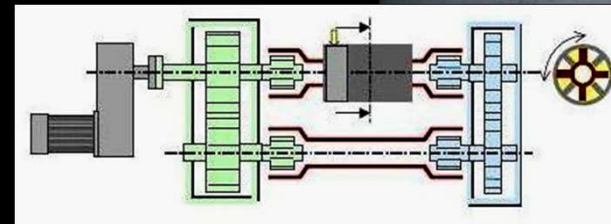
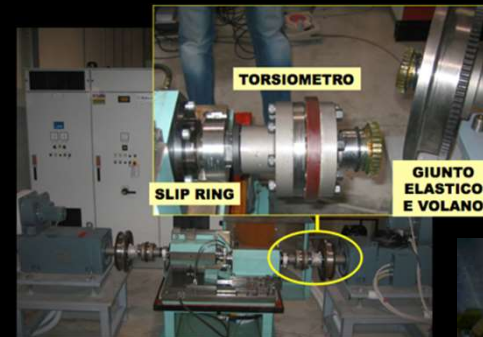
© 2005 Winston Mitchell

# Testing

- Test bench design
- Dynamic tests
- Endurance test (pitting fatigue)



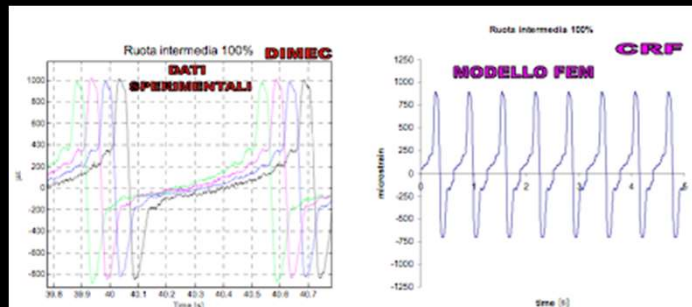
## PROBLEMATICHE NVH





# Testing

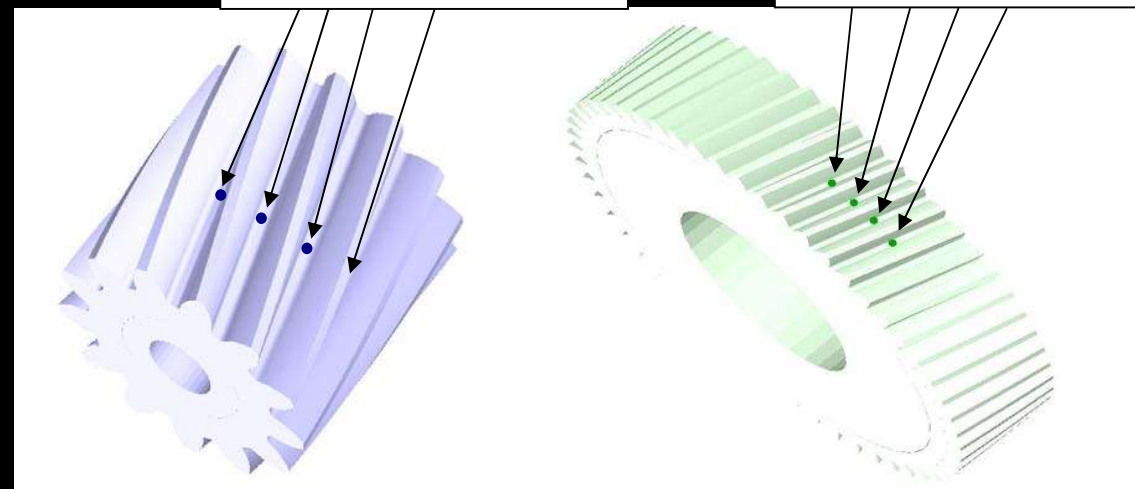
- Tooth-based deformation measurement under operating conditions
- Comparison with FEM models



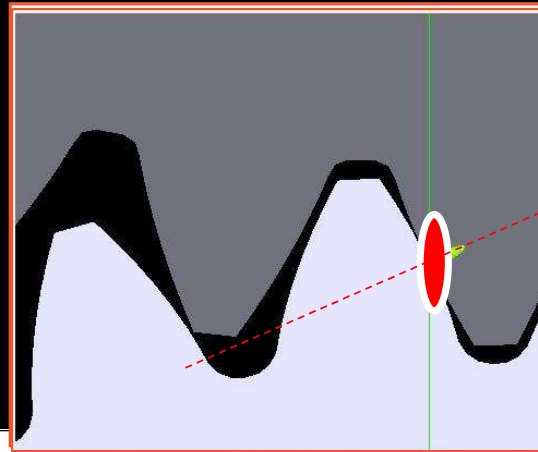
## PROBLEMATICHE NVH

4 estensimetri sul pignone

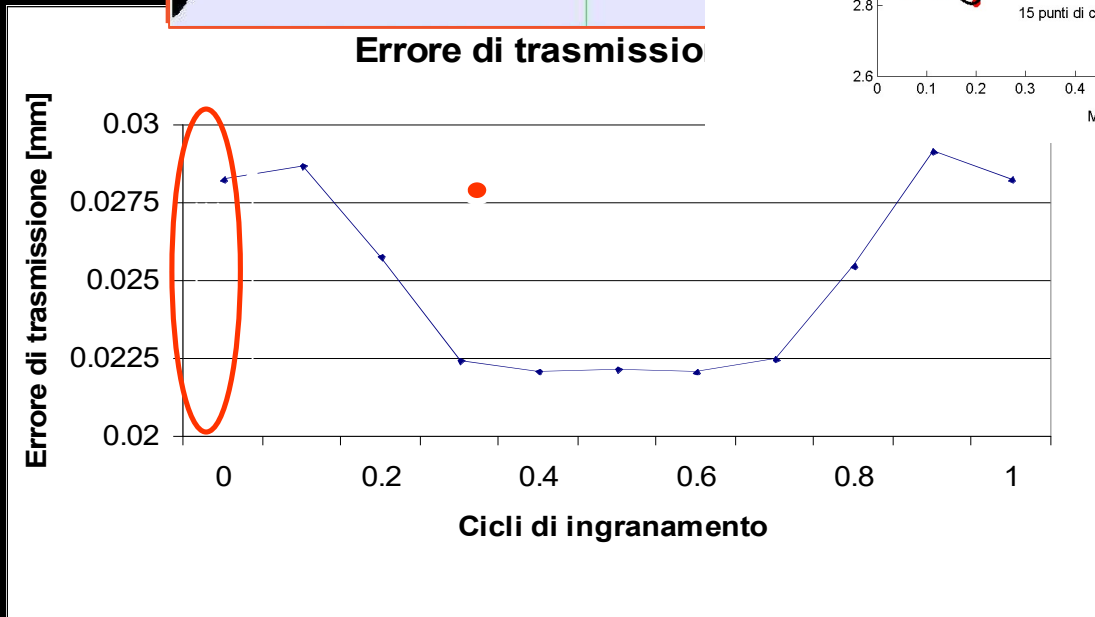
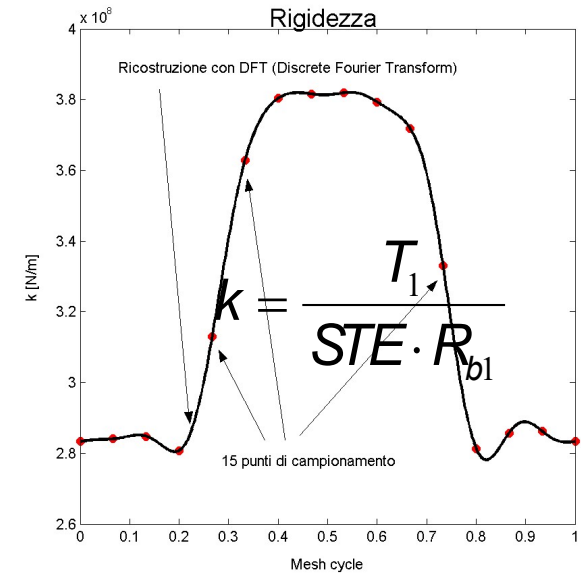
4 estensimetri su ruota lenta



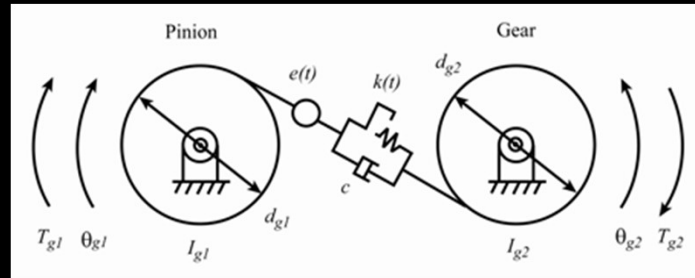
# TRANSMISSION ERROR AND VIBRATION



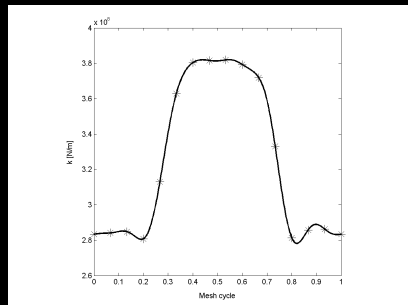
Errore di trasmissione



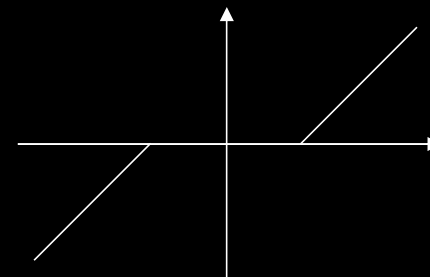
# DTE AND DYNAMIC MODEL



TIME VARIANT

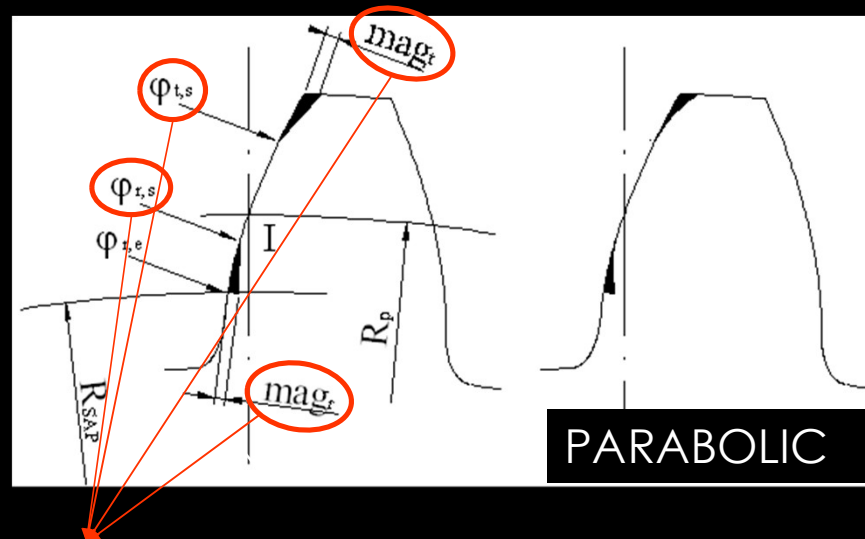


NON LINEAR



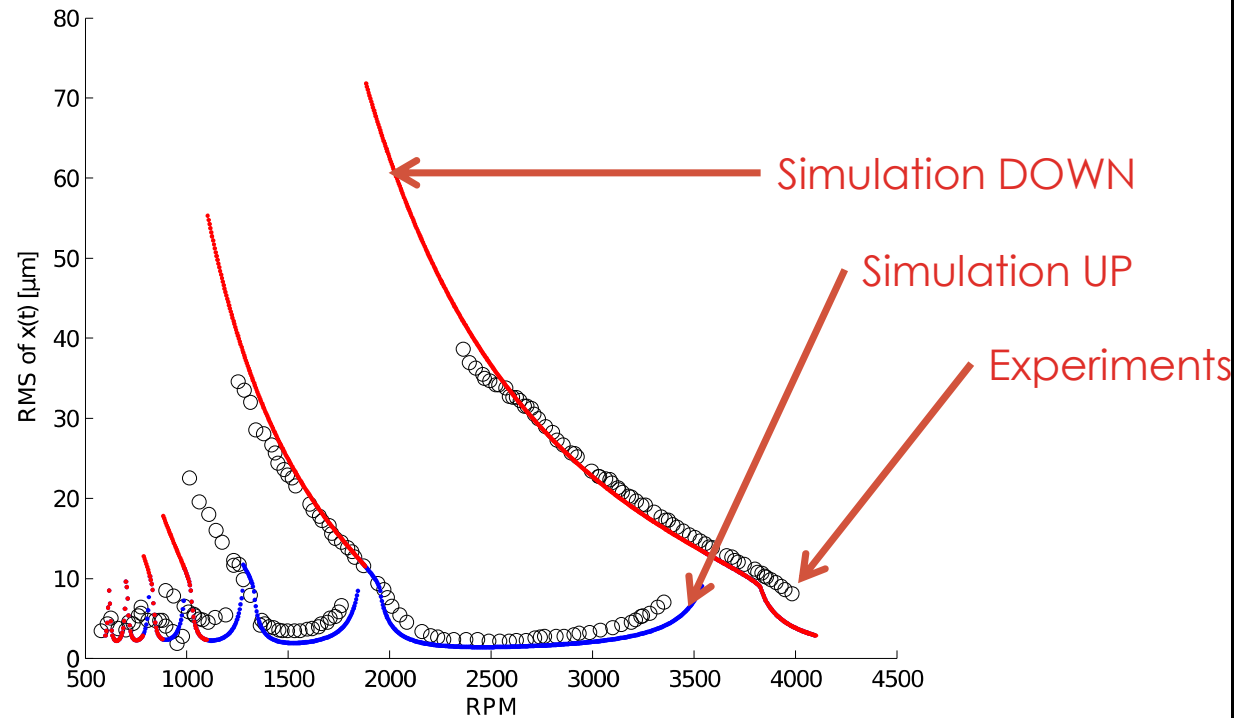
# DTE AND DYNAMIC MODEL

Profile modifications



$$m_e \ddot{x}(t) + c(\dot{x}(t) - \dot{a}(t)) + k(t) f(x(t) - a(t)) = T_g(t)$$

# MODEL VALIDATION



$$m_e \ddot{x}(t) + c(\dot{x}(t) - \dot{e}(t)) + k(t) f(x(t) - e(t)) = T_g(t)$$

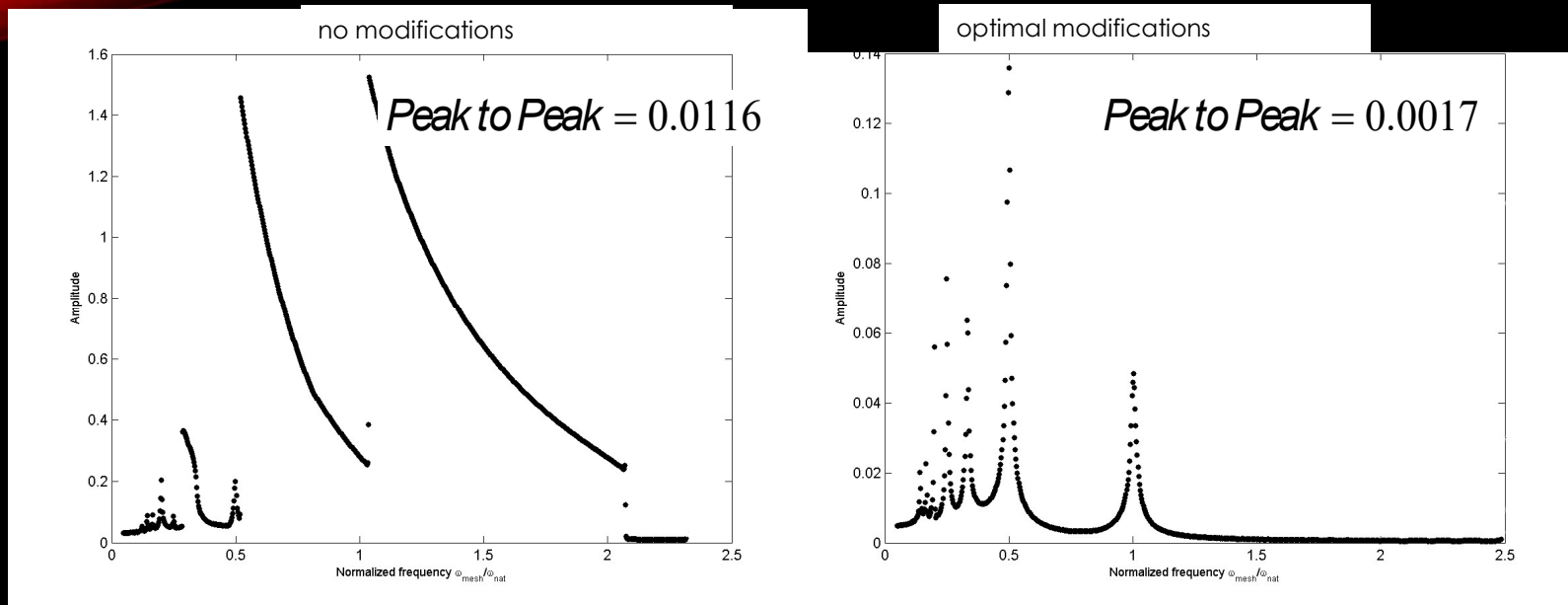




NVH ISSUES

# OPTIMIZATION

# DTE AND DYNAMIC MODEL



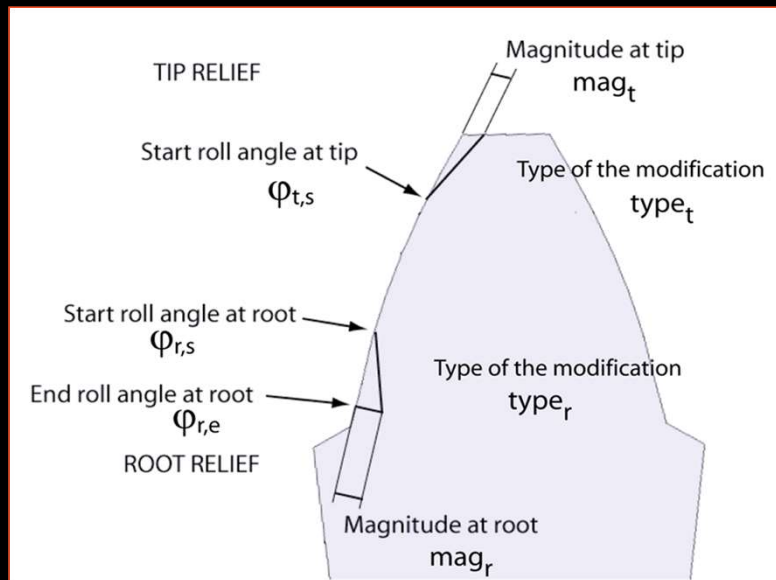
## EFFECT OF STATIC TRANSMISSION ERROR REDUCTION

- Maximum width reduced by ten times
- Fully linear behavior (without contact loosing)
- No parametric resonance
- Superharmonics more important than resonance at the natural frequency

# DEVELOPMENT OF OPTIMIZATION METHODS

To reduce the vibration of a gear, we act on the static transmission error to reduce its peak to peak:

1. Acting on the surface finish
2. Acting on macrogeometric parameters: conduct ratio, clearance, number of teeth
3. Practicing micrometric reliefs to the profile in the head and foot and on the side of the tooth



- **Modification parameters:**
- **Crowning values of the flanks**
- **Roll angle of the beginning of the reliefs in the tip and root of the profiles**
- **Extent of the reliefs**

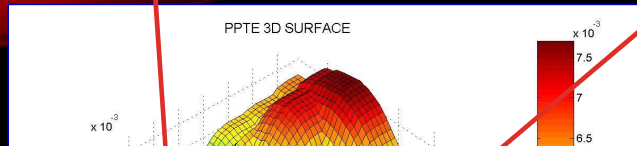
# OPTIMIZATION APPROACHES

- **BRUTE FORCE:** the whole parameter space is spanned
  - **A global minimum is found**
  - Heavy computational cost
  - Small parameter spaces can be spanned
- **EURISTIC:** two parameters are considered at each step
  - **The computational cost is reduced**
  - The optimum is not the minimum
  - The optimum depends on the initial set of parameters
- **GENETIC ALGORITHMS:** simulates the evolution process
  - **Finds a global minimum**
  - **The computational cost is reduced**
- **STOCHASTIC+SIMPLEX:**
  - **The Stochastic approach generates random set of parameters. The global minimum is found after a large number of steps.**
  - **The simplex method is robust, but local.**
  - **The combination Stochastic+Simplex gives good results**

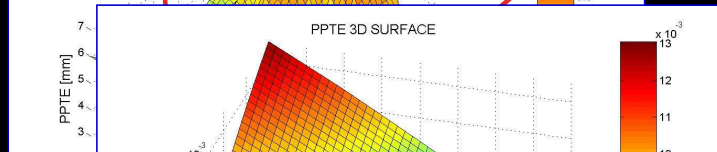
# ITERATIVE STATIC OPTIMIZATION

Minimum neglected: high gradients

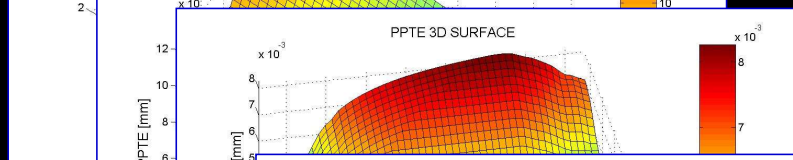
Optimal engineering



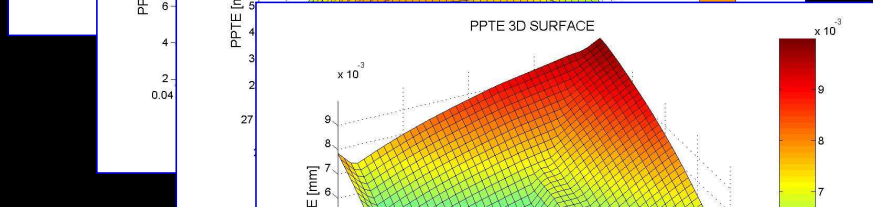
Minimum compared to the roll angle of start change in head on pinion and wheel



Minimal compared to the amplitude of changes in the head on pinion and wheel



Minimum compared to the roll angle of the start of modification to the foot on pinion and wheel



Minimal compared to the amplitude of the foot modifications on pinion and wheel

The optimal configuration is obtained:

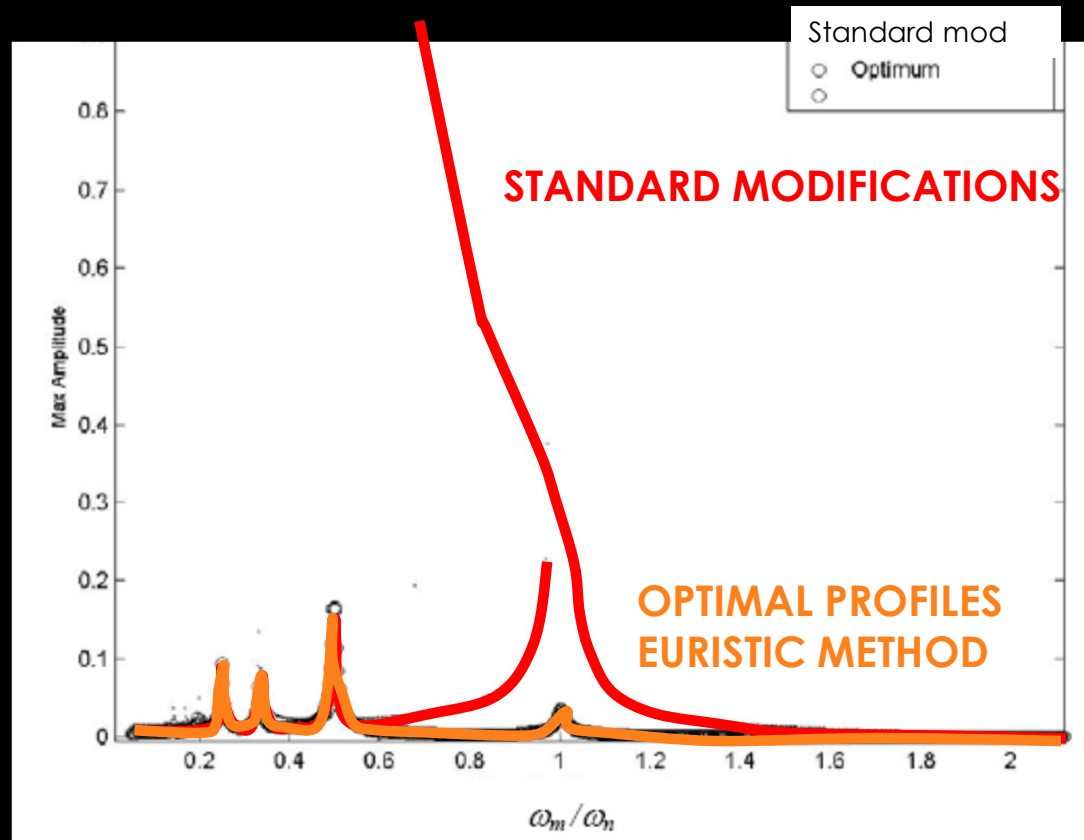
In the case under consideration peak to peak reduced from 0.0051 mm to 0.00198 mm (-61%)

ng configuration

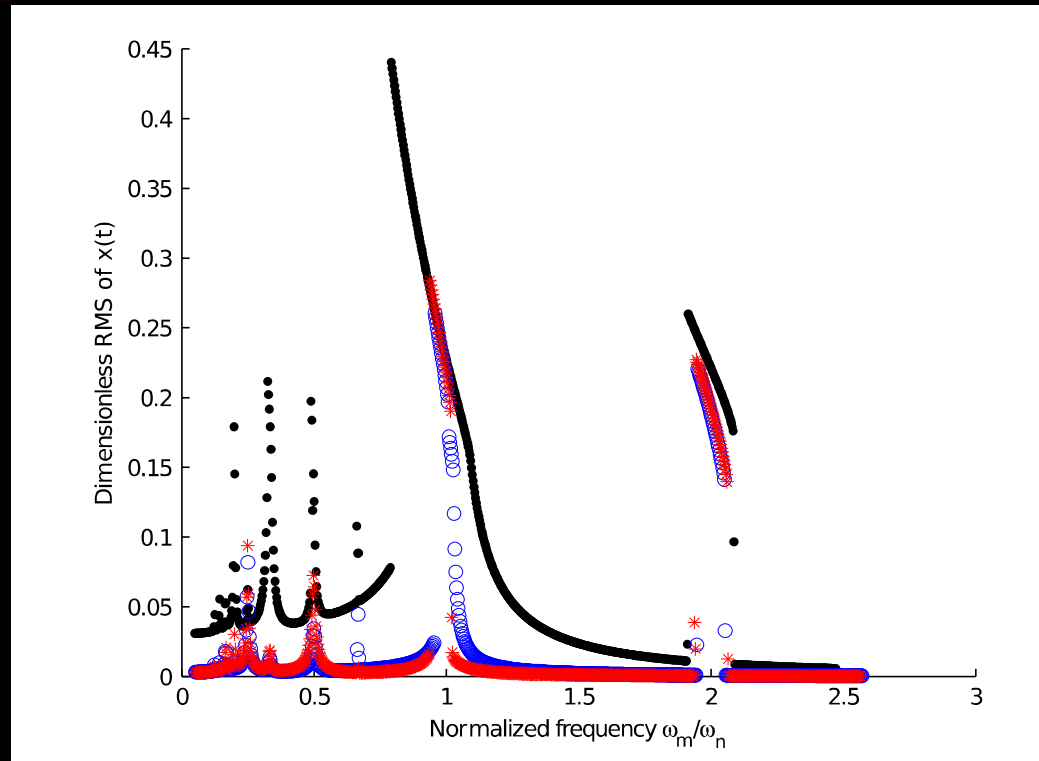
l configuration



# EURISTIC OPTIMIZATION: DYNAMICS



# MODELLING TECHNIQUES: OPTIMIZATION



OVERALL VIBRATION  
REDUCTION  
AFTER GENETIC  
ALGORITHM OPT  
REASONABLY HIGH  
COMPUTATIONAL COST

on UNIMORE-CNH)

on UNIMORE-CNH)

ean project)

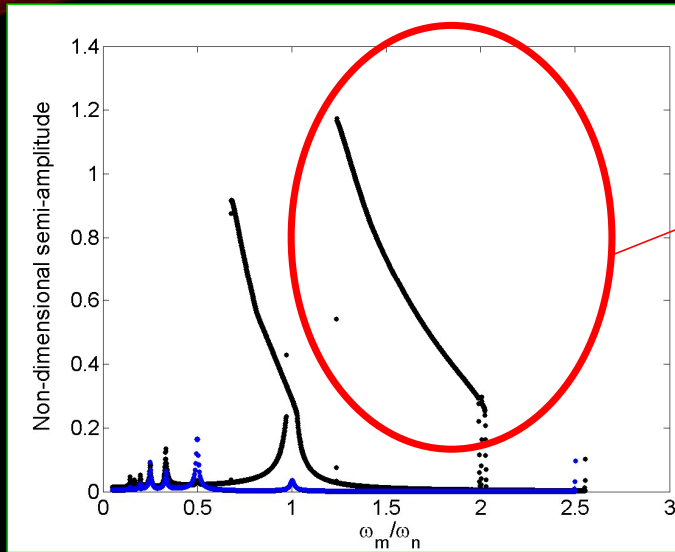
arboxes opt)

G. Bonori, M. Barbieri and F. Pellicano, "Optimum Profile Modifications of Spur Gears by Means of Genetic Algorithms", J. of Sound and Vibration, **313** (2008) 603–616.

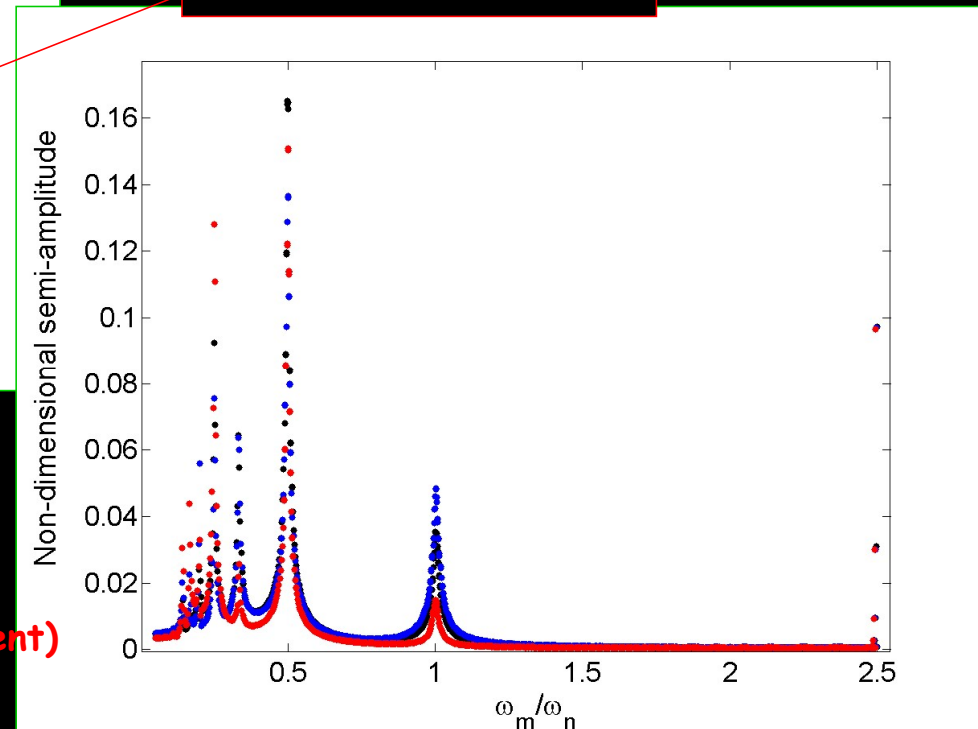
# EURISTIC VERSUS GENETIC OPTIMIZATION

BLACK: INITIAL GEARS  
BLUE: EURISTIC

PARAMETRIC INSTABILITY



BLACK: EURISTIC  
BLUE: GENETIC (PPSTE)  
RED: GENETIC (harmonic content)



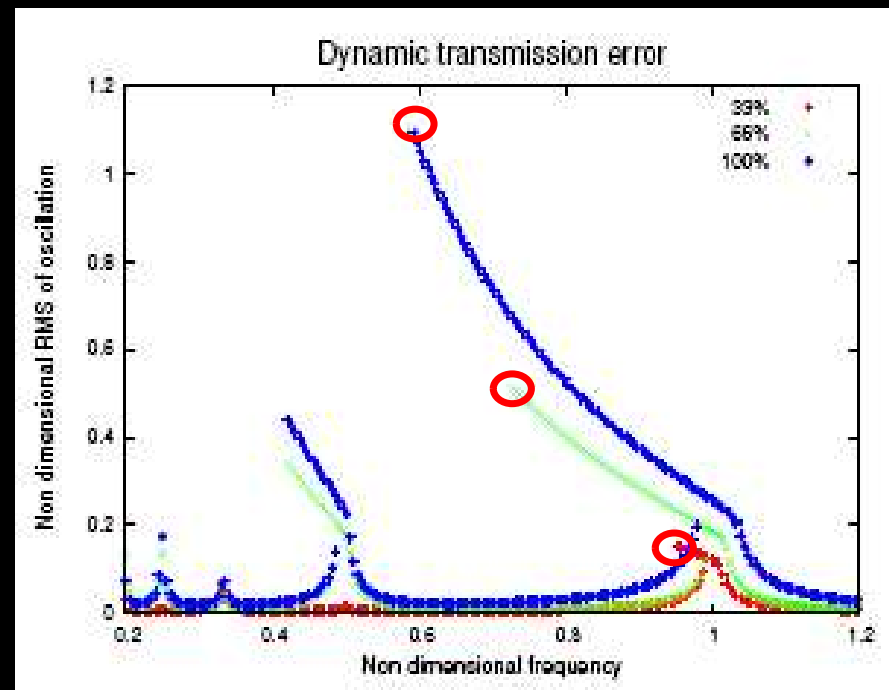
**GENETIC ALGORITHM: GLOBAL OPTIMIZATION METHOD, 8 PARAMETERS**

# DYNAMIC OPTIMIZATION

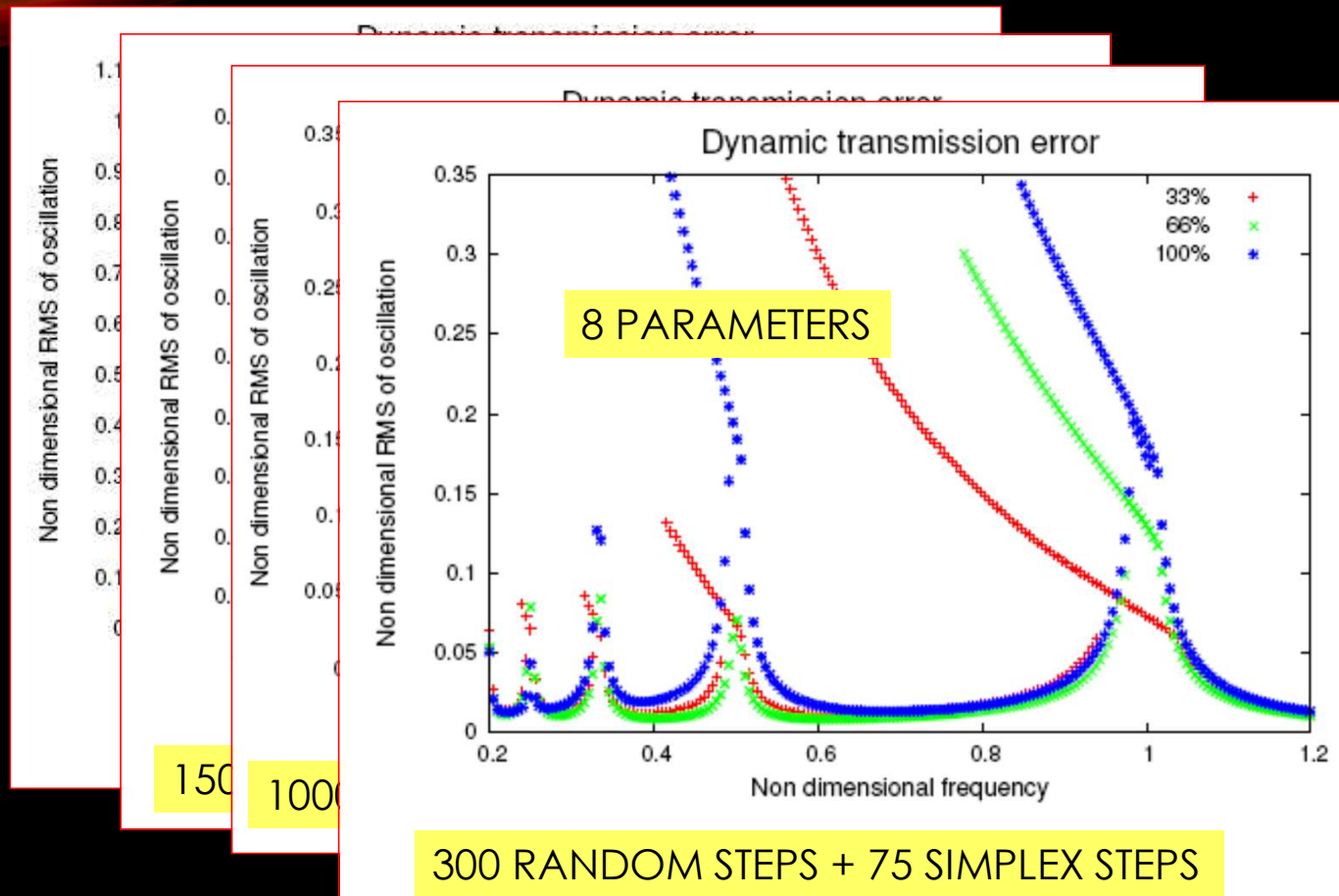
THE DYNAMICS CHANGES AS THE TRANSMITTED TORQUE VARIES

THE OBJECTIVE FUNCTION IS RELATED TO THE DYNAMIC SCENARIO

- MAX AMPLITUDE
- AVERAGE RMS
- **MAXIMUM RMS**
- ....



# DYNAMIC OPTIMIZATION





# COMPUTATIONAL COST

EVALUATION OF THE STATIC TRANSMISSION ERROR (or  $k(t)$ )

**15 or more nonlinear FEM analyses**

## STATIC OPTIMIZATION

**EURISTIC:**  $4 \times 25^2 = 2500$  PPTE evaluations;  $2500 \times 15 = 37500$  FEM analyses

**GENETIC ALGORITHMS** 50 cases (population)  $\times$  100 iterations = 5000 PPTE eval.

**75000 FEM ANALYSES**

## DYNAMIC OPTIMIZATION

**RANDOM (1000) + SIMPLEX (150)** 1150 STE evaluations: **17250 FEM ANALYSES**

200 time histories for each dynamic scenario

**230000 time histories**



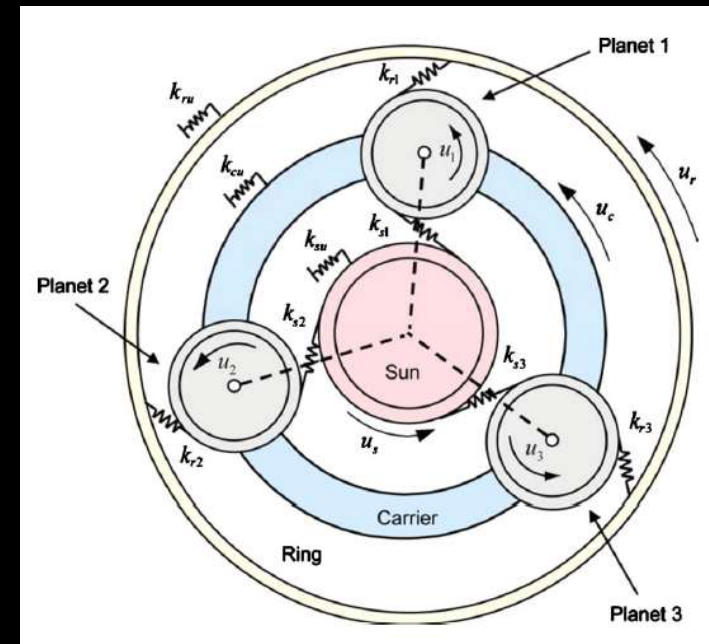
NVH ISSUES

# PLANETARY DYNAMIC MODELS

# DYNAMIC MODELS: PLANETARY



- Concentrated parameter models
- Equivalent stiffness from nonlinear FEM analyses
- Linear models and invariant time: proper frequencies, modes
- Linear time-variant models: global static transmission error
- Nonlinear models: non-stationary dynamics and broadband responses (noise)



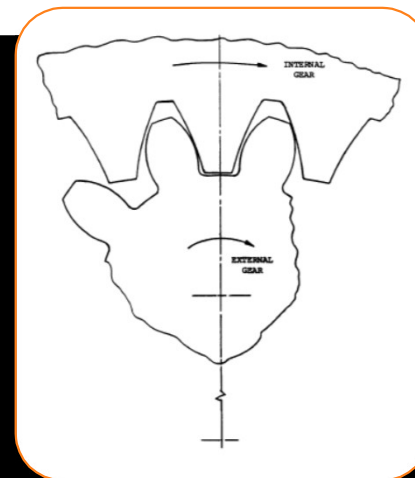
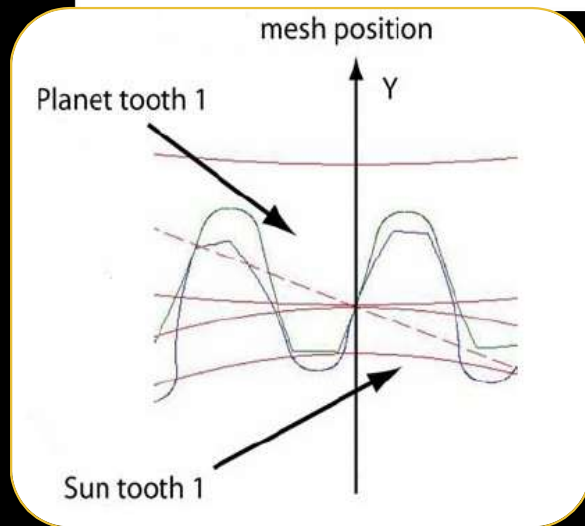
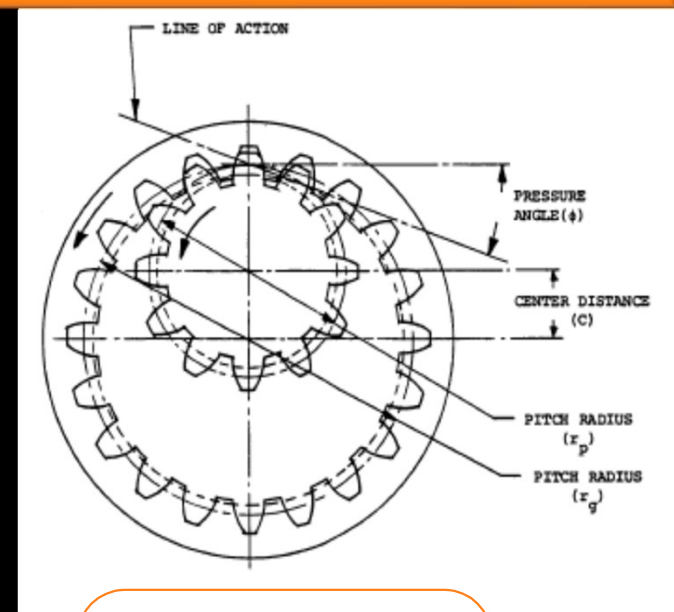
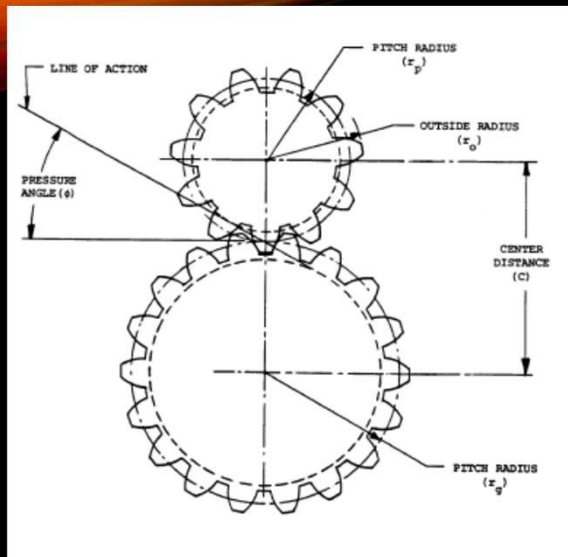
# SUN-PLANET AND RING-PLANET MESH STIFFNESS

1. Gear geometry generation.
2. Automatic meshing of gears (FEM model and mounting).
3. Generation of an output, compatible with a FEM software (MSC/marc).
4. Launching calculations for a desired load case and extracting results in terms of meshing stiffness.

**The following results have been obtained using the GearDesign software**

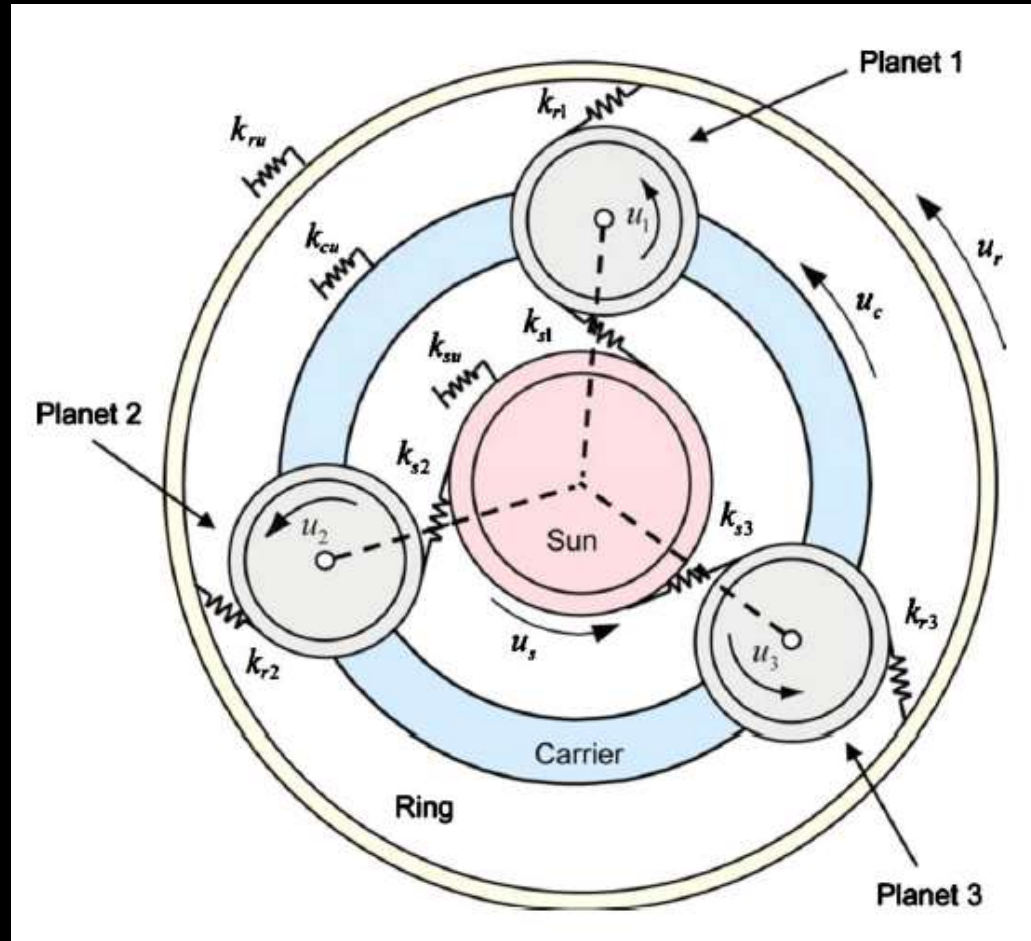
# Ring-Planet Mesh

NVH ISSUES



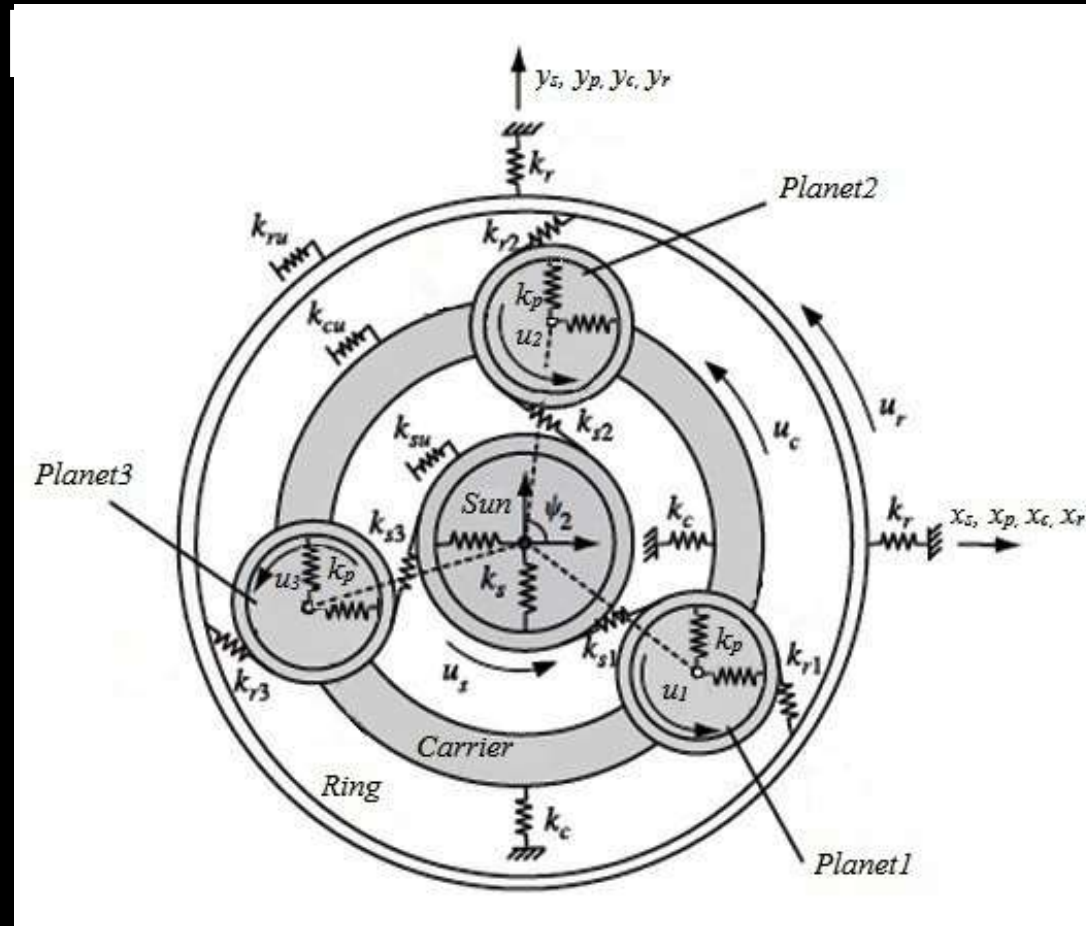


**CONCENTRATED  
PARAMETER MODEL  
ROTATIONAL DEGREES  
ONLY**



**AMIC  
ODEL**

- LUMPED PARAMETER MODEL
- ROTATIONAL AND TRANSLATIONAL DEGREES
- TAKES INTO ACCOUNT THE COMPLIANCE OF THE BEARINGS



# SUN GEAR EQUATIONS

Inertia Bearing Viscosity

Bearing Stiffness

$$-M_s \ddot{x}_s - C_s \dot{x}_s + \sum_{n=1}^N \left[ (c_{sn} (\dot{x}_n - \dot{x}_s) \cdot \sin^2(\psi_n - \alpha_s)) + (-c_{sn} (\dot{y}_n - \dot{y}_s) \cdot \sin(\psi_n - \alpha_s) \cdot \cos(\psi_n - \alpha_s)) + (c_{sn} (\dot{\theta}_s \cdot r_{bs} + \dot{\theta}_n \cdot r_{bn}) \cdot \sin(\psi_n - \alpha_s)) \right] - k_s x_s +$$

$x$   
Translational  
DOF

Sun-Planet Mesh viscosity

$$(k_{sn} (f_{s\theta}^{***}) \sin(\psi_n - \alpha_s)) = 0$$

Piecewise Linear Functions for simulating sun-planet backlash

$y$   
Translational  
DOF

$$\dot{y}_s) \cdot \cos^2(\psi_n - \alpha_s)) + (-c_{sn} (\dot{\theta}_s \cdot r_{bs} + \dot{\theta}_n \cdot r_{bn}) \cdot \cos(\psi_n - \alpha_s)) \left] - k_s y_s +$$

$$\sum_{n=1}^N \left[ (-k_{sn} (f_{sx}^*) \cdot \sin(\psi_n - \alpha_s) \cdot \cos(\psi_n - \alpha_s)) + (k_{sn} (f_{sy}^{**}) \cdot \cos^2(\psi_n - \alpha_s)) +$$

$$(-k_{sn} (f_{s\theta}^{***}) \cdot \cos(\psi_n - \alpha_s)) \right] = 0$$

Rotational  
DOF

$$T_s - I_s \ddot{\theta}_s - C_{su} \dot{\theta}_s + \sum_{n=1}^N \left[ (-c_{sn} (\dot{x}_n - \dot{x}_s) \cdot \sin(\psi_n - \alpha_s) \cdot r_{bs}) + (c_{sn} (\dot{y}_n - \dot{y}_s) \cdot \cos(\psi_n - \alpha_s) \cdot r_{bs}) + (-c_{sn} (\dot{\theta}_s \cdot r_{bs} + \dot{\theta}_n \cdot r_{bn}) \cdot r_{bs}) \right] - k_{su} \theta_s + \sum_{n=1}^N \left[ (-k_{sn} (f_{sx}^*) \cdot \sin(\psi_n - \alpha_s) \cdot r_{bs}) +$$

$$(k_{sn} (f_{sy}^{**}) \cdot \cos(\psi_n - \alpha_s) \cdot r_{bs}) + (-k_{sn} (f_{s\theta}^{***}) \cdot r_{bs}) \right] = 0$$

# PIECEWISE LINEAR FUNCTIONS FOR SIMULATING SUN-PLANET BACKLASH

$$f_{sx} = \begin{cases} x_n - x_s - \frac{b_s}{\sin(\psi_n - \alpha_s)} & \Delta_s \geq b_s \longrightarrow \text{Contact} \\ 0 & |\Delta_s| < b_s \longrightarrow \text{Tooth Separation} \\ x_n - x_s + \frac{b_s}{\sin(\psi_n - \alpha_s)} & \Delta_s \leq -b_s \longrightarrow \text{Backside Contact} \end{cases}$$

$$f_{sy} = \begin{cases} y_n - y_s + \frac{b_s}{\cos(\psi_n - \alpha_s)} & \Delta_s \geq b_s \longrightarrow \text{Contact} \\ 0 & |\Delta_s| < b_s \longrightarrow \text{Tooth Separation} \\ y_n - y_s - \frac{b_s}{\cos(\psi_n - \alpha_s)} & \Delta_s \leq -b_s \longrightarrow \text{Backside Contact} \end{cases}$$

$$f_{s\theta} = \begin{cases} \theta_s \cdot r_{bs} + \theta_n \cdot r_{bn} - b_s & \Delta_s \geq b_s \longrightarrow \text{Contact} \\ 0 & |\Delta_s| < b_s \longrightarrow \text{Tooth Separation} \\ \theta_s \cdot r_{bs} + \theta_n \cdot r_{bn} + b_s & \Delta_s \leq -b_s \longrightarrow \text{Backside Contact} \end{cases}$$

$$\Delta_s = [(x_n - x_s) \cdot \sin(\psi_n - \alpha_s) - (y_n - y_s) \cdot \cos(\psi_n - \alpha_s) + (\theta_s \cdot r_{bs} + \theta_n \cdot r_{bn})]$$

# SYSTEM EQUATIONS

- THE EQUATIONS MUST BE REPLICATED FOR
- All satellites
- The ring
- The train carrier
- For a three-satellite system we obtain a system of 18 NON-SMOOTH (nonlinear) time-variant differential equations (the ring is locked)



# VALIDATION

Comparisons with Cheon-Jae Bahk, R.G. Parker (2011) Journal of Computational and Nonlinear Dynamics “**Analytical Solution for the Nonlinear Dynamics of Planetary Gears**”

Parameters of the case study planetary gear set

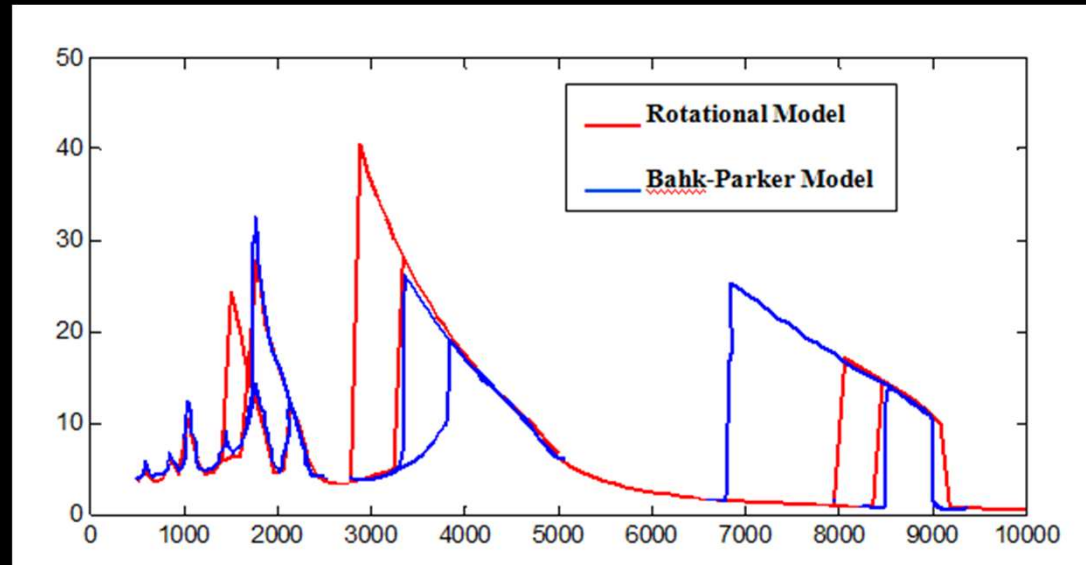
Parameter	Sun	Ring	Planet	Carrier
Number of teeth	27	99	35	-
Module (mm)	2.8677	2.7782	2.8677	-
Pressure angle (deg)	24.60	20.19	24.60	-
Working Center Distance (mm)		88.89		-
Root diameter (mm)	70.485	284.150	91.440	-
Outer diameter (mm)	84.074	304.800	105.004	-
Inner diameter (mm)	57.15	271.73	73.66	-
Base diameter (mm)	70.40	258.130	91.26	177.80
Translational bearing stiffness (N/m)	-	-	-	-
Rotational bearing stiffness (N.m/Rad)	0	2.19e9	0	2.19e9
$I/r^2$ (Kg)	3.11	4.89		24.80
Mass (Kg)		2.64		

# COMPARISON OF NATURAL FREQUENCIES FOR PURE ROTATIONAL MODEL

Natural Freq (Hz)	Rotational model	Bahk-Parker model
$\omega_1$	1847	1846
$\omega_3 = \omega_2$	760	2744
$\omega_4$	4387	4379

Cheon-Jae **Bahk**, R.G. **Parker** , 2011. "Analytical Solution for the Nonlinear Dynamics of Planetary Gears"

## COMPARISON OF THE RMS OF SUN ROTATION (AFTER SUBTRACTION OF THE MEAN VALUE)



The resonance peaks lean to the left, implying **softening nonlinearity** induced by tooth separation.

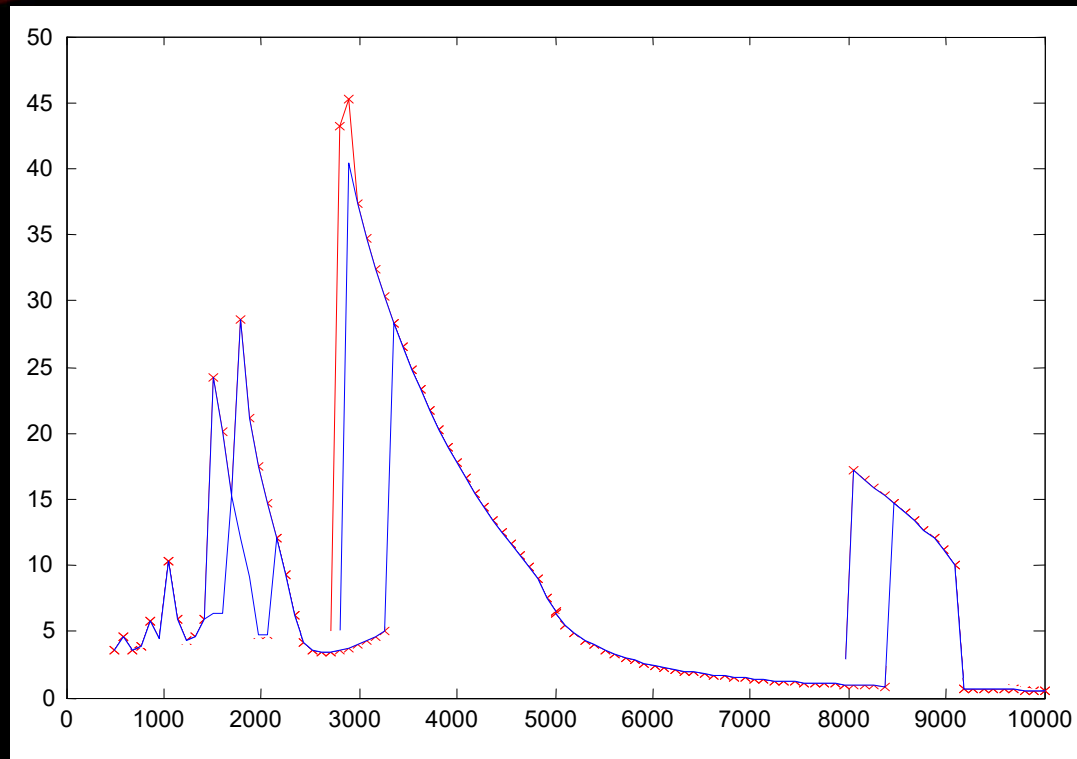
There are additional resonance peaks around 8000 Hz, 1800 Hz the first distinct mode, and below 1600 Hz mesh frequency. These peaks are the combined effects of parametric instability from higher harmonics of mesh stiffness variation and **nonlinear subharmonic and superharmonic** resonances of the first and second distinct modes.

# COMPARISON OF NATURAL FREQUENCIES FOR ROTATIONAL-TRANSLATIONAL MODEL

Natural Freq (Hz)	Rotational model	Bahk-Parker model
$\omega_1 = \omega_2$	1758	1760
$\omega_3$	2091	2095
$\omega_4 = \omega_5$	3352	3390
$\omega_6$	5245	5249

Cheon-Jae **Bahk**, R.G. **Parker** , 2011. "Analytical Solution for the Nonlinear Dynamics of Planetary Gears"

## COMPARISON OF THE PURE ROTATIONAL MODEL (BLUE LINE) WITH ROTATIONAL-TRANSLATIONAL MODEL (RED LINE)



Pure rotational model has 4 DOF (rotation of sun and three planets)  
and the second model has 12 DOF (one rotation and two translation  
for sun and each planet)



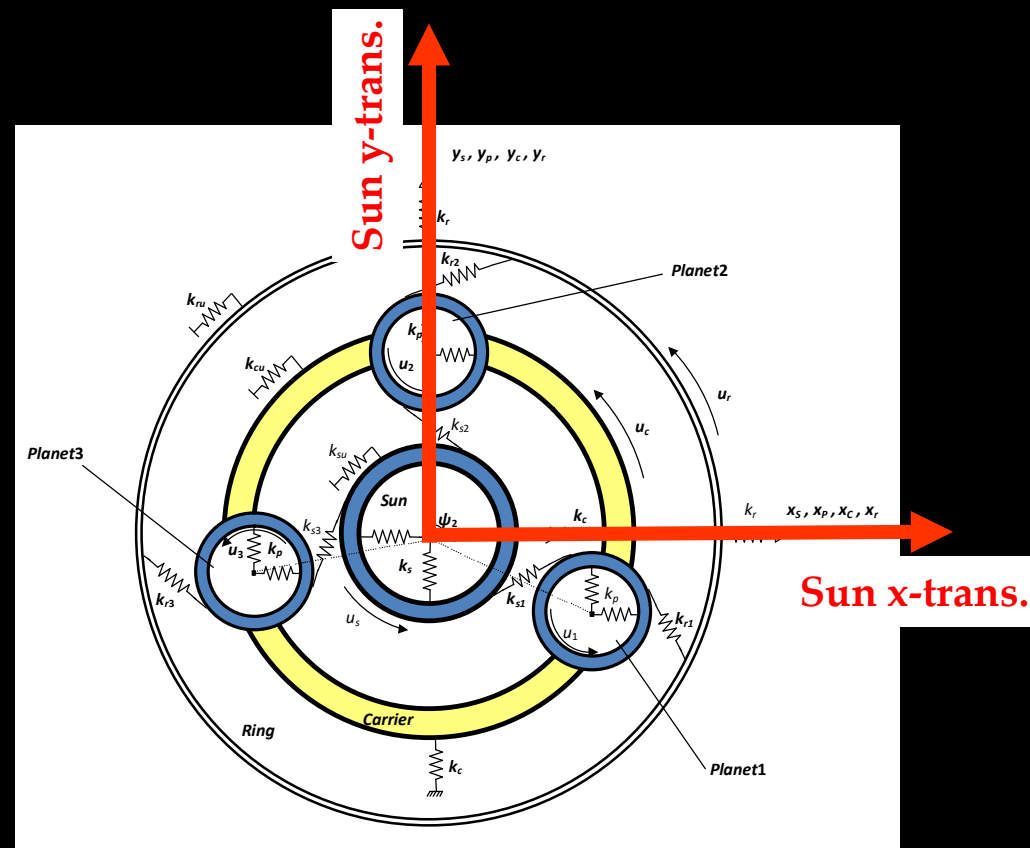


# **PLANETARY GEARS**

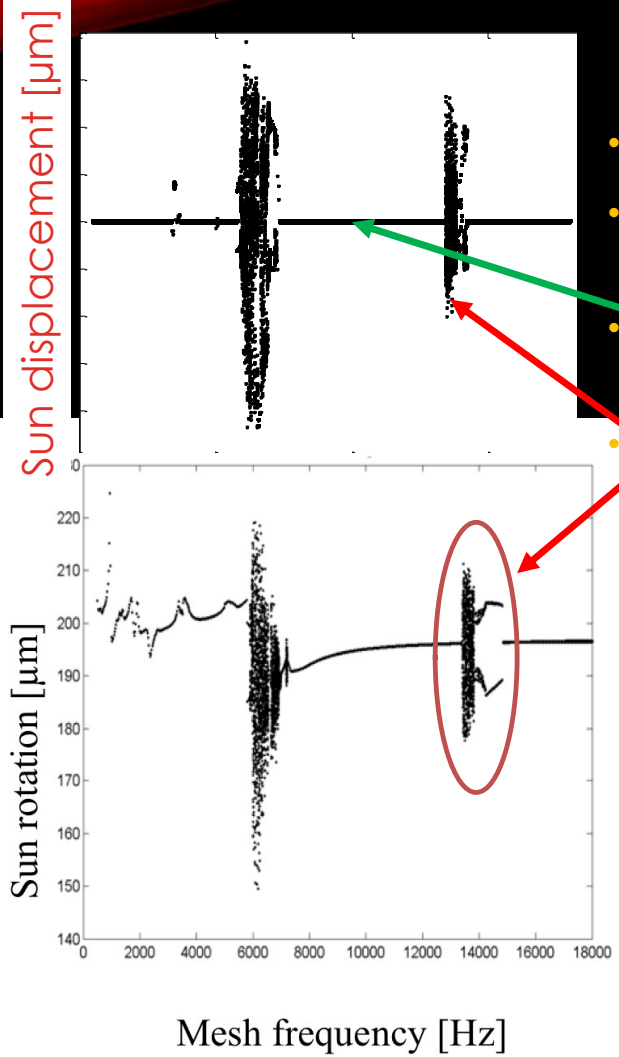
## **COMPLEXITY AND RELIEFS OPTIMIZATION**

# Dynamic imbalance on the sun (and compensation)

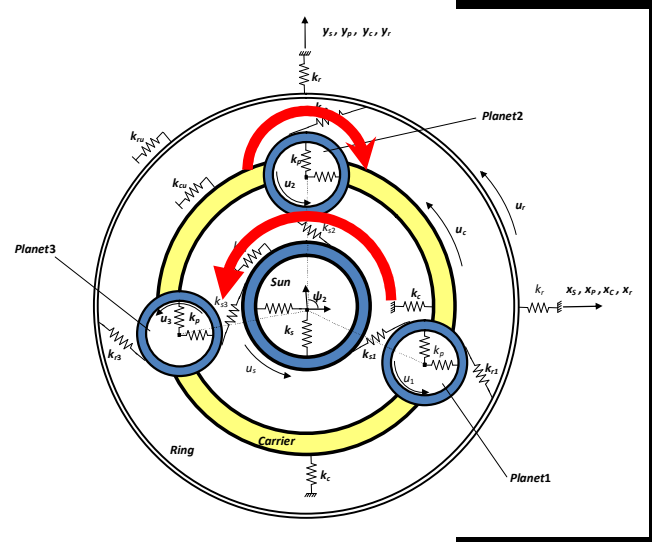
- With 3 planets placed at 120 degrees and phased, forces acting on the sun should be balanced
- When chaos occurs, the sun can lose balancing



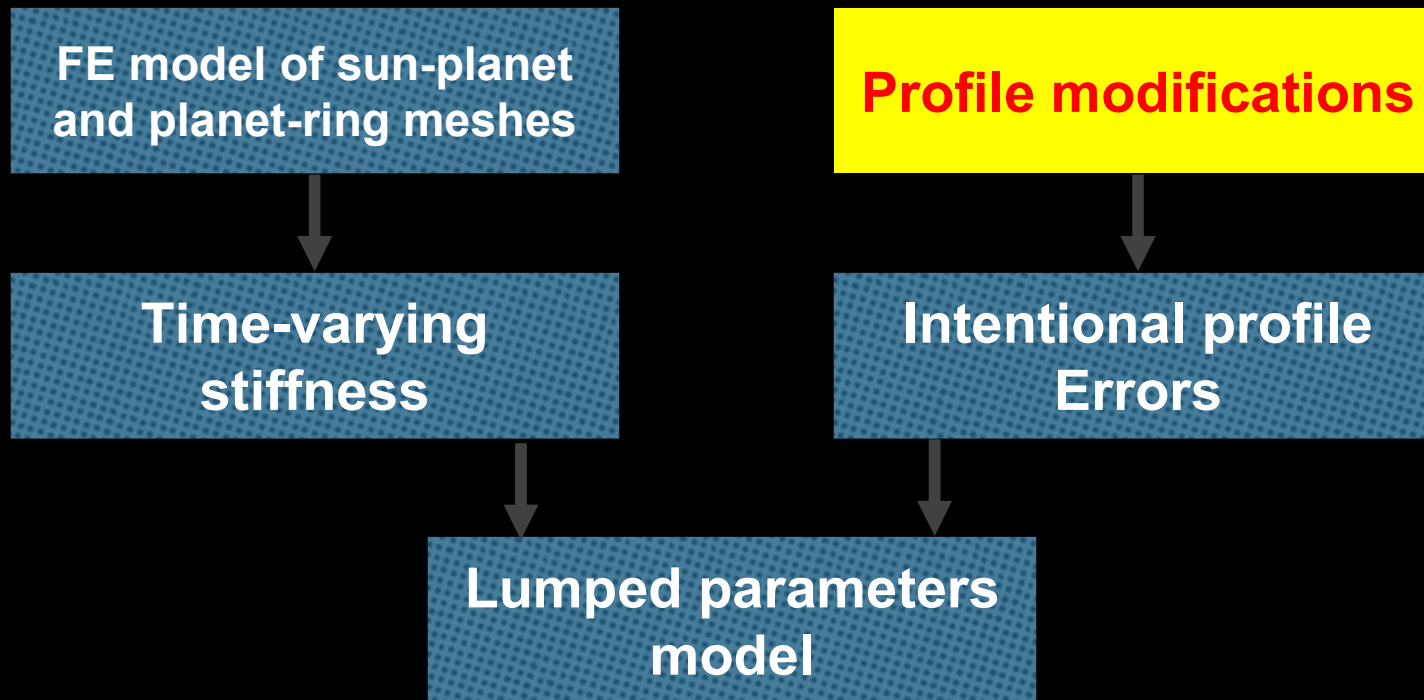
# Bifurcation diagrams



- Rotation reduced along the line of action  $\theta \times R_{base}$
- Non smooth period doubling bifurcation and chaos
- Sun displacement should be zero due to perfect balancing and phasing
- Chaos induces imbalance



# DYNAMIC MODEL OF A PLANETARY GEAR WITH MODIFICATIONS (COMPENSATION)



- The first step is modeling both meshes by FEM, using our software HPGA
  - Later on, rigid rotations due to profile modifications are taken into account

where,

# APPLYING TOOTH PROFILE MODIFICATIONS

## Displacement Function's Correction

piecewise-linear displacement functions for sun-planets meshing:

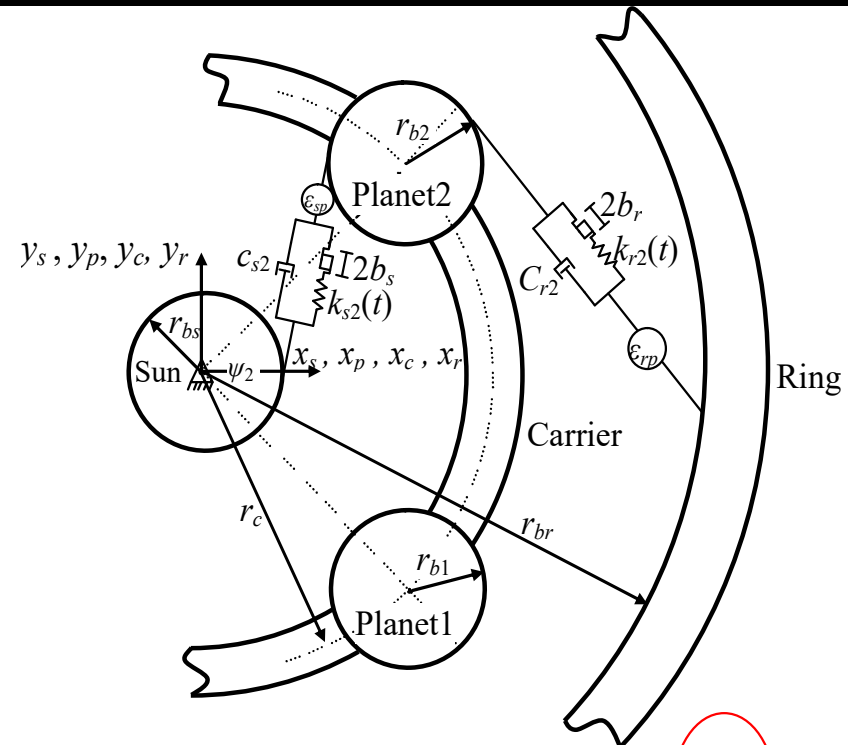
$$f_{sx} = \begin{cases} x_n - x_s - \frac{\varepsilon_{sp}}{\sin(\psi_n - \alpha_s)} & \Lambda > 0 \\ 0 & \Lambda = 0 \\ x_n - x_s + \frac{-\varepsilon_{sp} + 2b_s}{\sin(\psi_n - \alpha_s)} & \Lambda < 0 \end{cases}$$


---


$$f_{sy} = \begin{cases} y_n - y_s - \frac{\varepsilon_{sp}}{\cos(\psi_n - \alpha_s)} & \Lambda > 0 \\ 0 & \Lambda = 0 \\ y_n - y_s - \frac{(2b_s - \varepsilon_{sp})}{\cos(\psi_n - \alpha_s)} & \Lambda < 0 \end{cases}$$


---

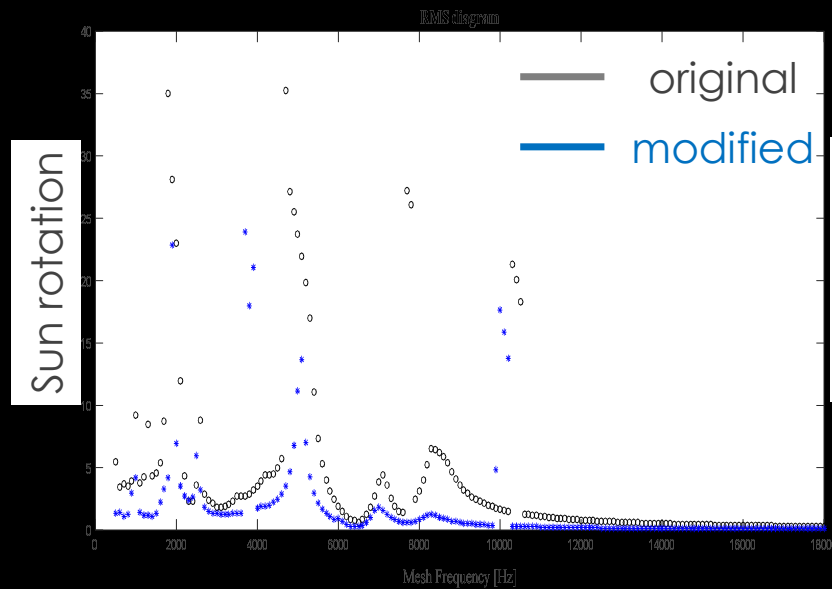

$$f_{st} = \begin{cases} \theta_s \cdot r_{bs} + \theta_n \cdot r_{bn} - \varepsilon_{sp} & \Lambda > 0 \\ 0 & \Lambda = 0 \\ \theta_s \cdot r_{bs} + \theta_n \cdot r_{bn} - \varepsilon_{sp} - 2b_s & \Lambda < 0 \end{cases}$$



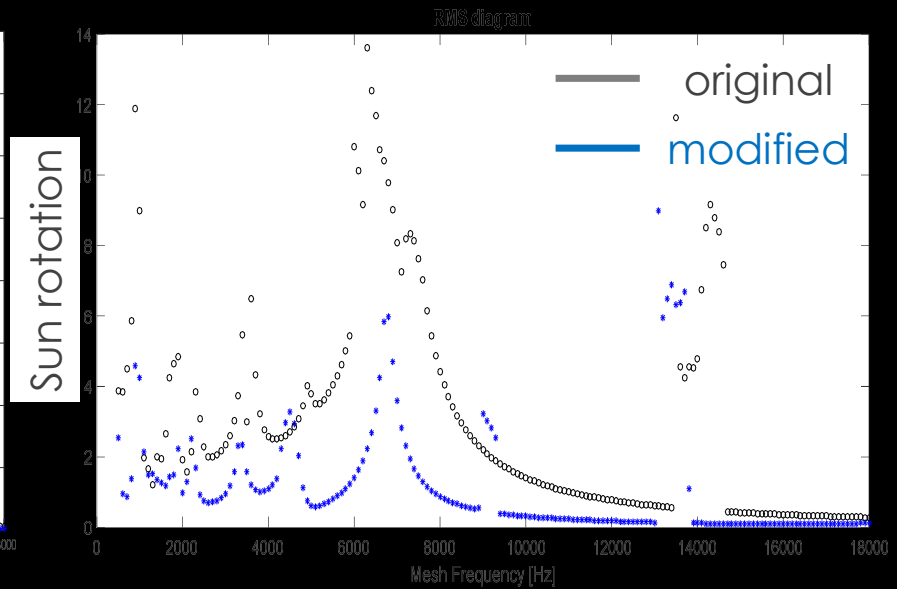


# EFFECT OF PROFILE MODIFICATIONS

## VIBRATION AMPLITUDE-FREQUENCY



(Planet translational bearing stiffness is equal to  $2.19e9$  N/m)

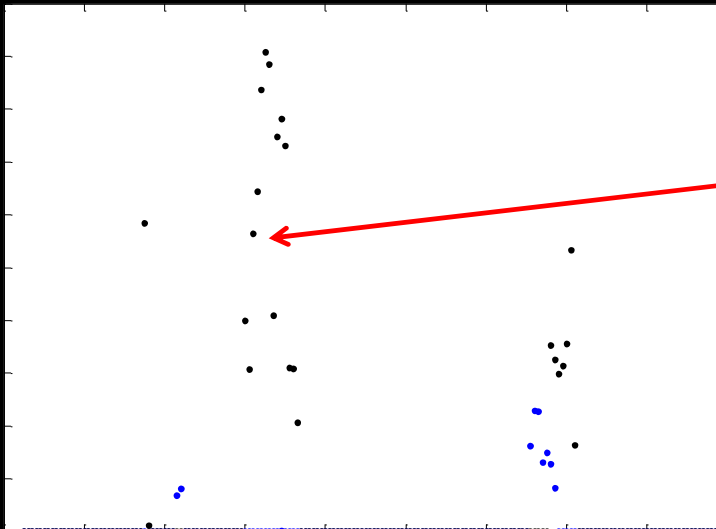


(Planet translational bearing stiffness is equal to  $2.19e8$  N/m)

# DYNAMIC IMBALANCE ON THE SUN (AND COMPENSATION)

## VIBRATION AMPLITUDE

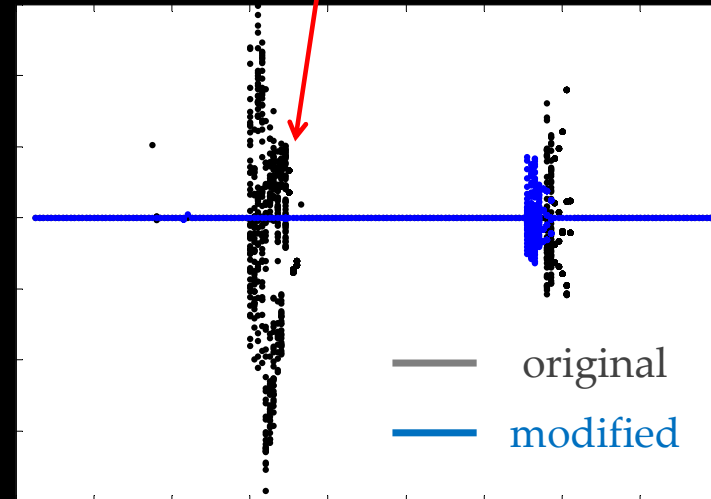
Sun translation (x-dir.)



CHAOS INDUCES IMBALANCE

## BIFURCATION DIAGRAM

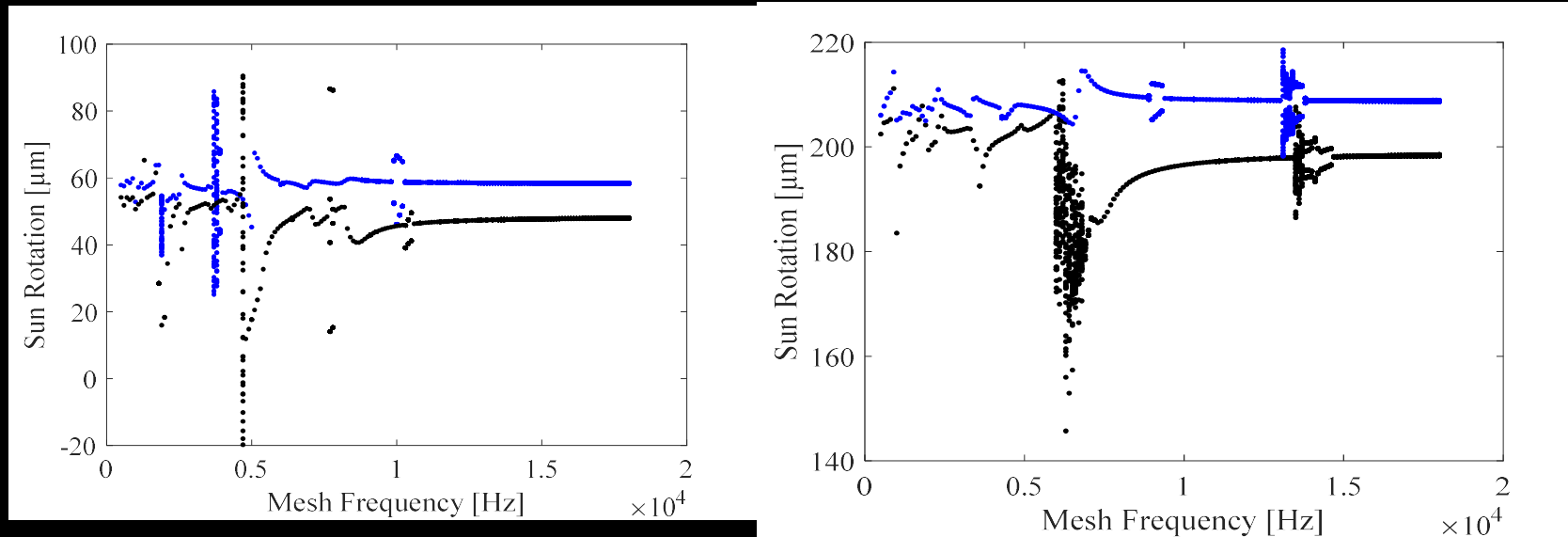
Sun translation (x-dir.)



PROFILE MODIFICATIONS CAN REDUCE THE CHAOS INDUCED IMBALANCE

# EFFECT OF PROFILE MODIFICATION

## BIFURCATION DIAGRAMS



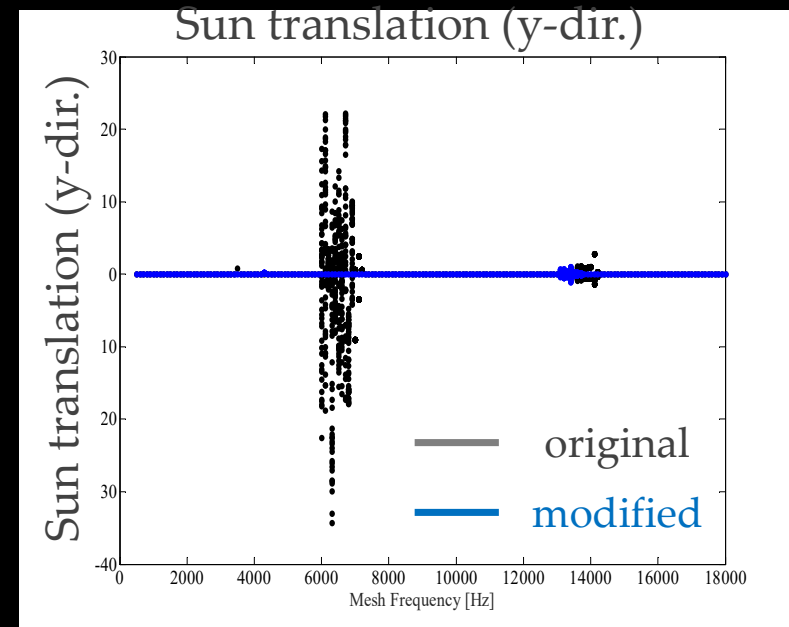
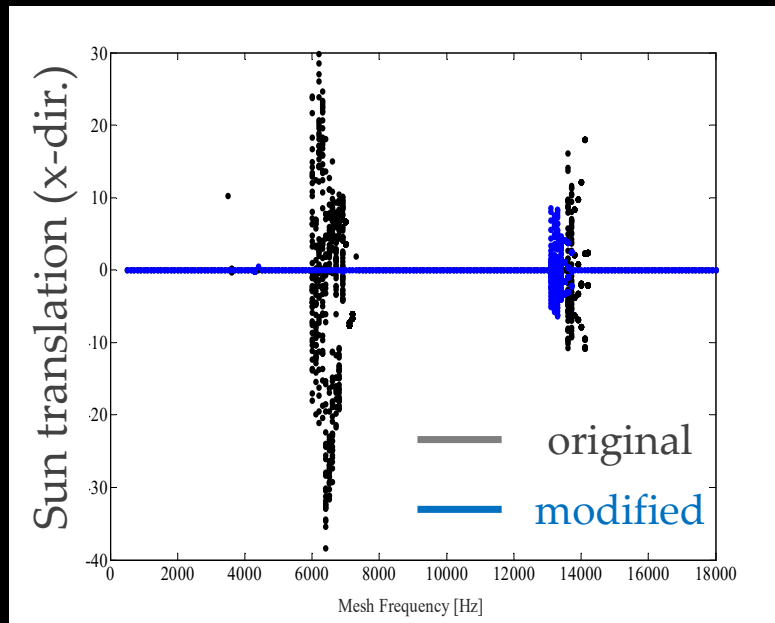
(Planet translational bearing stiffness  
is equal to  $2.19 \times 10^9$  N/m)

(Planet translational bearing stiffness  
is equal to  $2.19 \times 10^8$  N/m)

- The bifurcation diagrams show that instability regions are smaller for modified gears
- Instability disappears for the resonance at 6900 Hz

# Dynamic imbalance on the sun (and compensation)

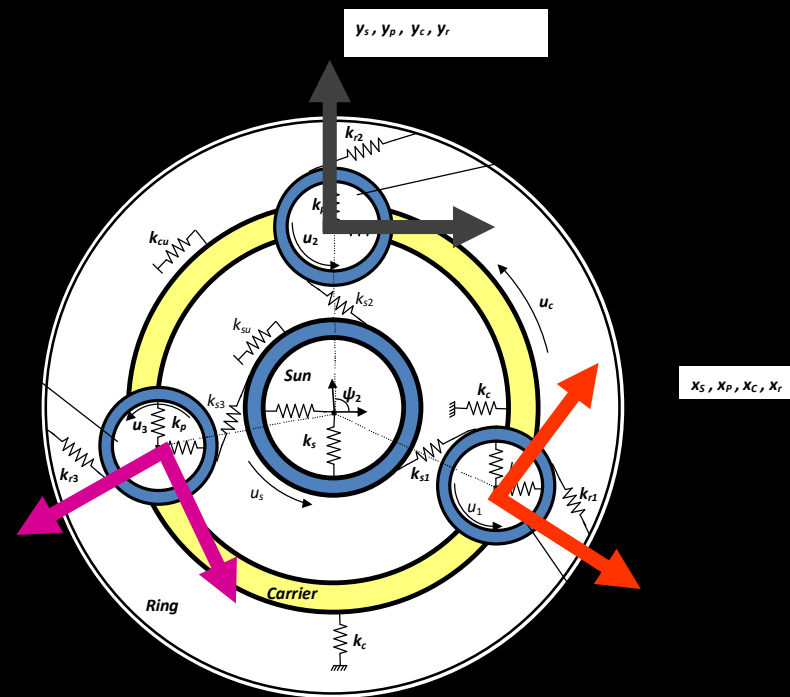
## BIFURCATION DIAGRAMS



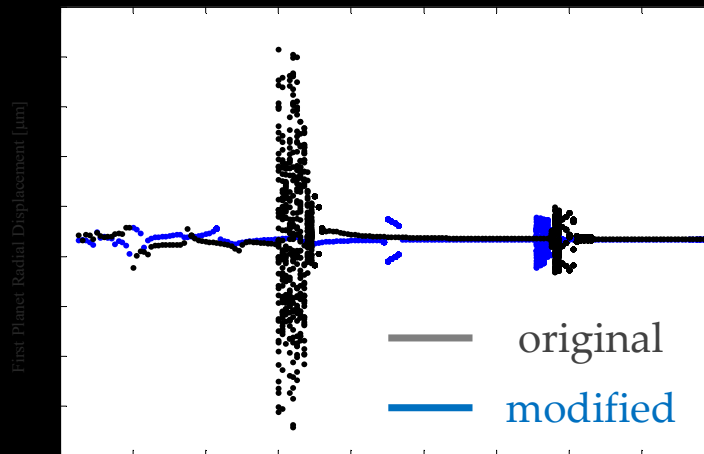
- With 3 planets placed at 120 and phased, forces acting on the sun should be balanced
- When chaos occurs, the sun is no longer balanced

# PLANETS RADIAL DISPLACEMENT

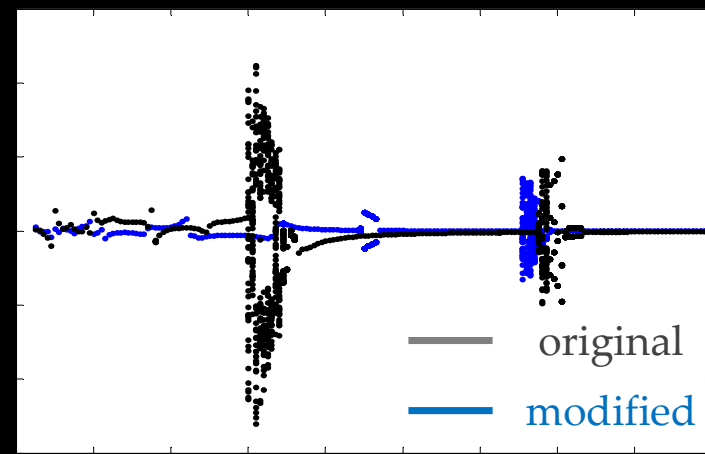
## Radial and tangential displacements



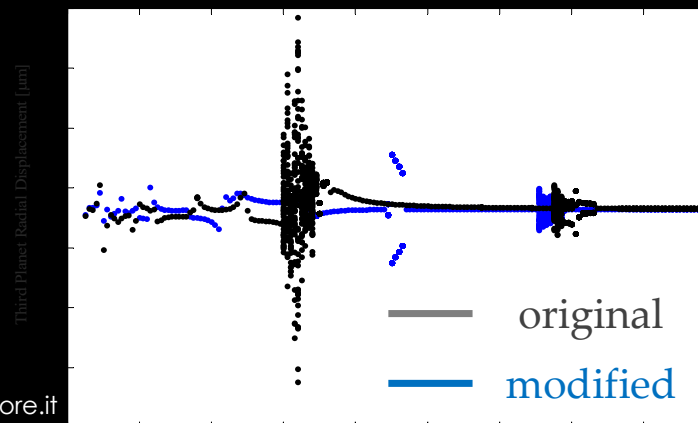
# PLANETS RADIAL DISPLACEMENT



Mesh Frequency [Hz]



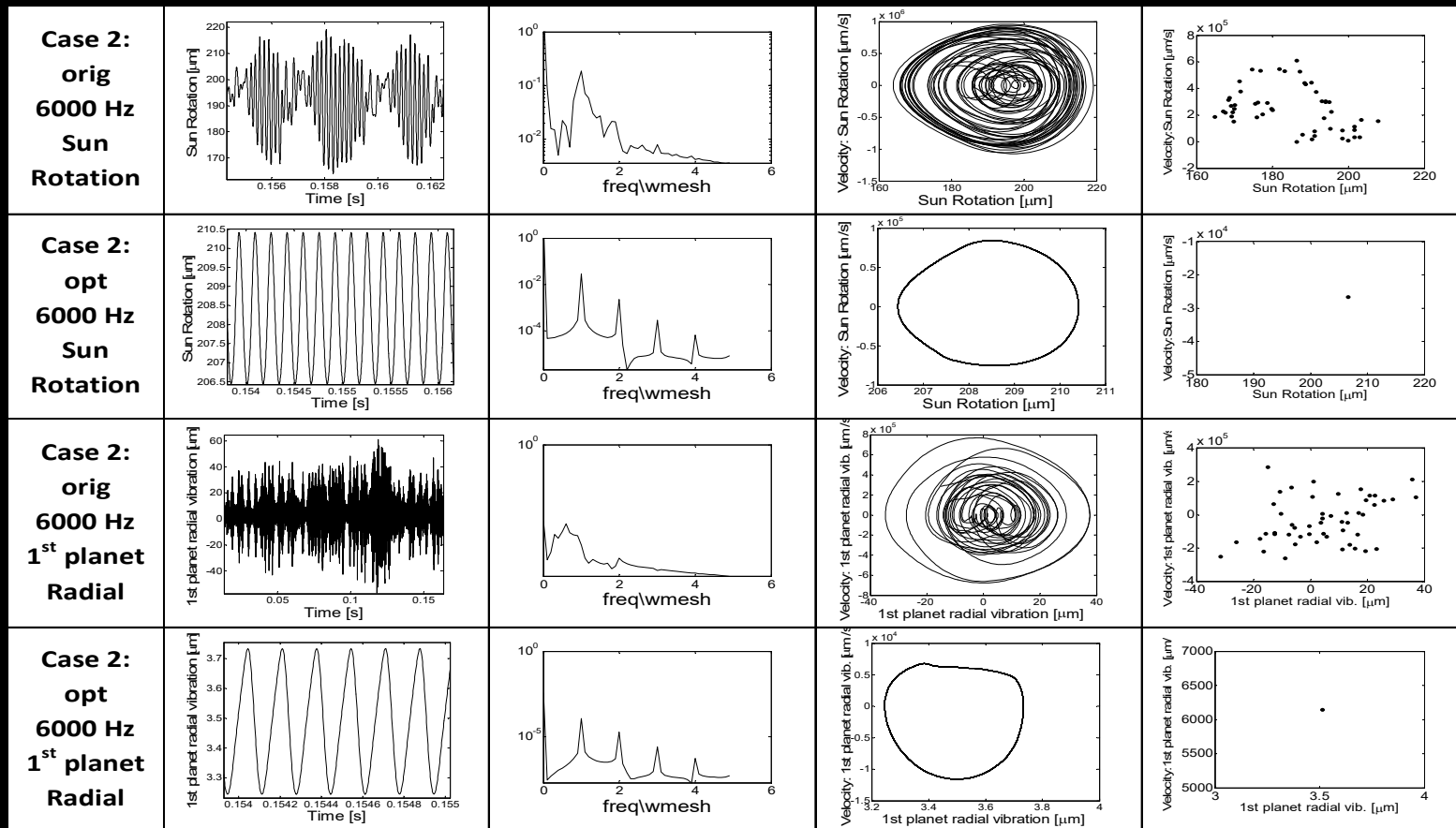
Mesh Frequency [Hz]



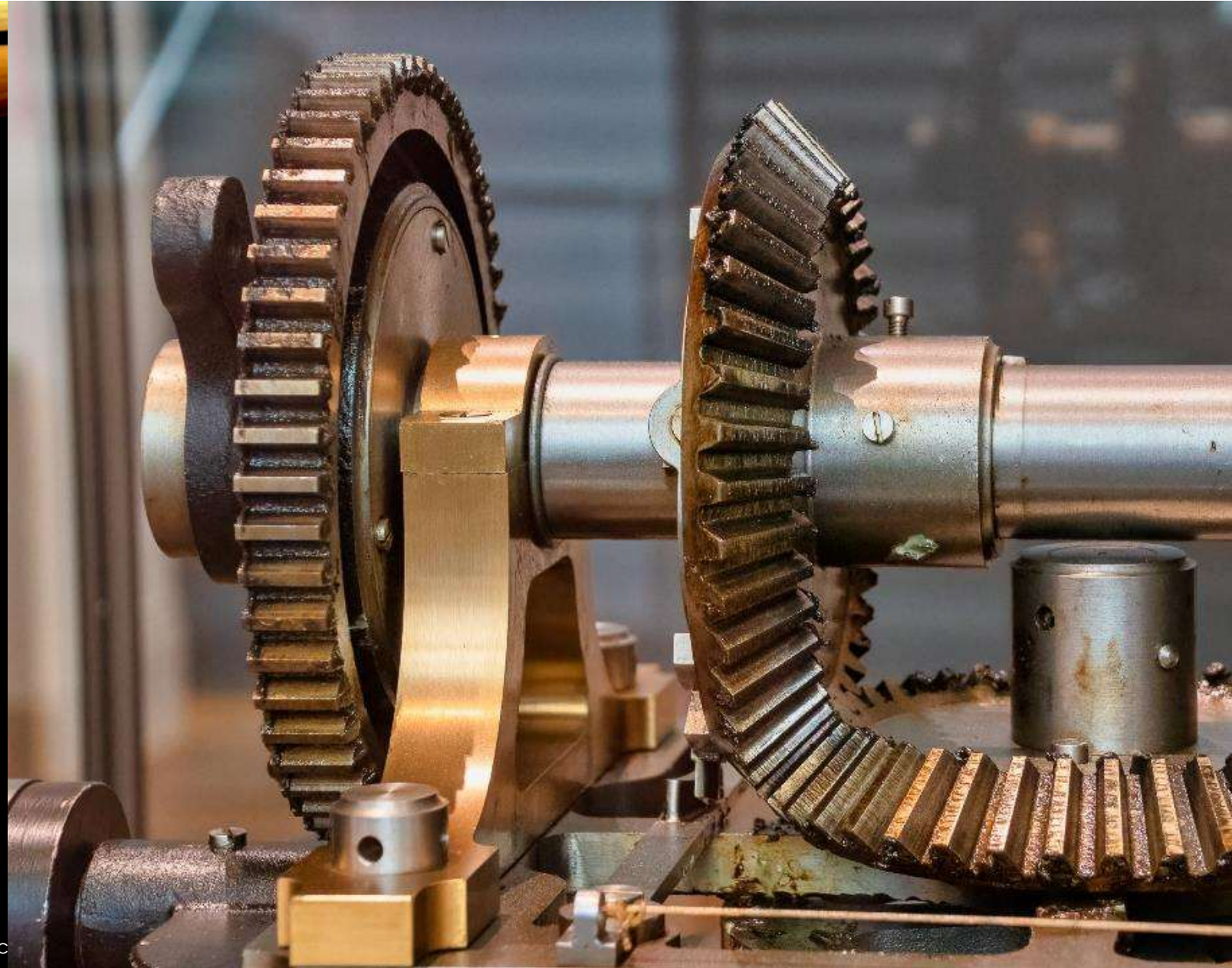
Mesh Frequency [Hz]



# EFFECT OF MODIFICATION



# TESTING FACILITIES



F. Pellicano – A. Zippo - francesco.pellicano@unipi.it



## Test Rig 1:

### Goal:

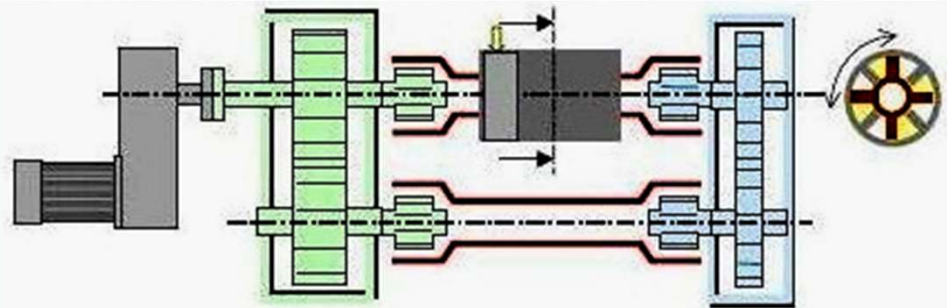
- Endurance Analysis
- Stress measurement

### Software:

ANSOL. Transmission3D

### System:

Back-to-back configuration



# Test Rig 1: Validation

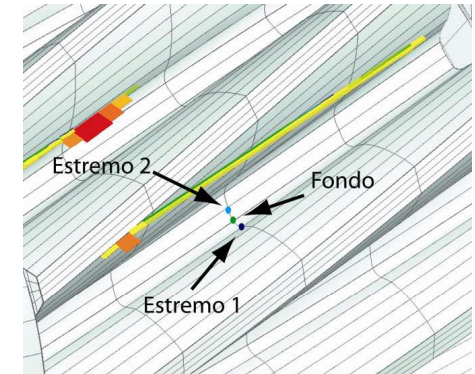
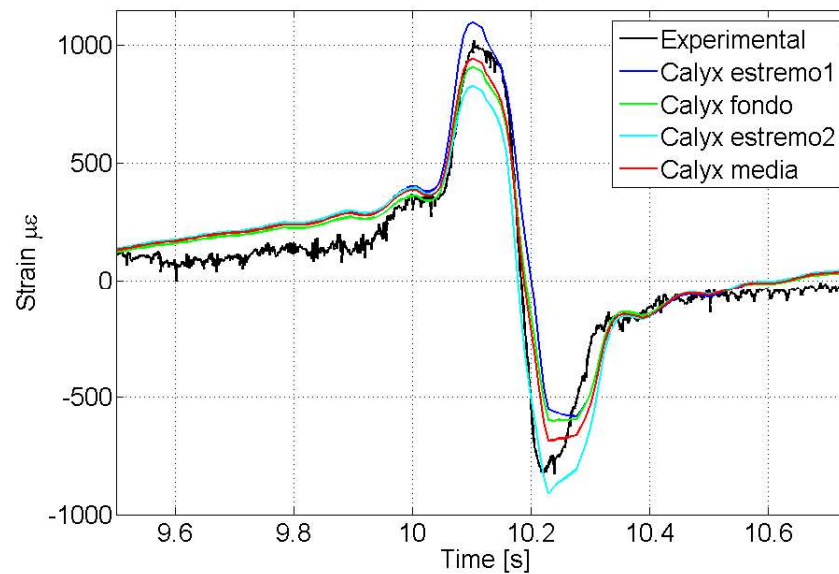
## Conducted comparison:

- Simulation based on FEM
- Experimental test

## Goal:

Defining Accuracy

FEM-based Software:  
ANSOL. Transmission3D



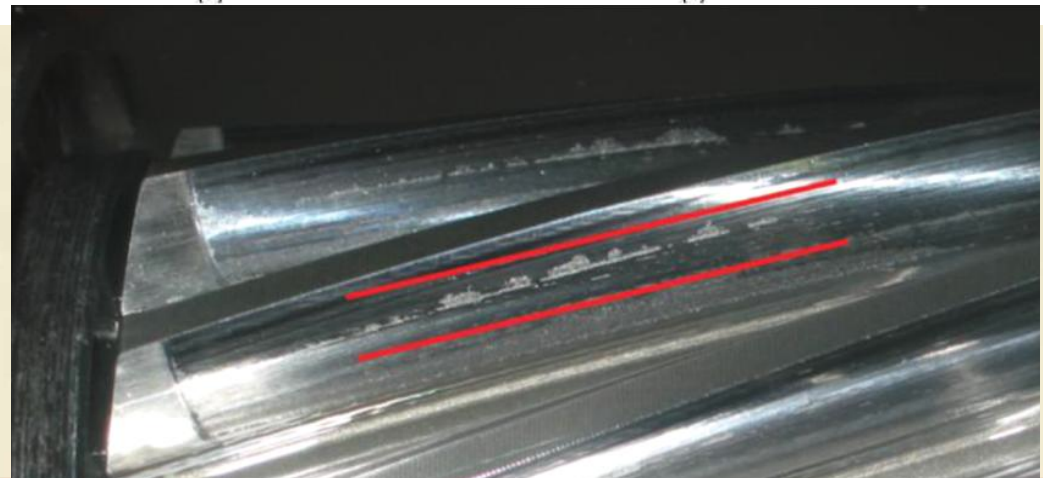
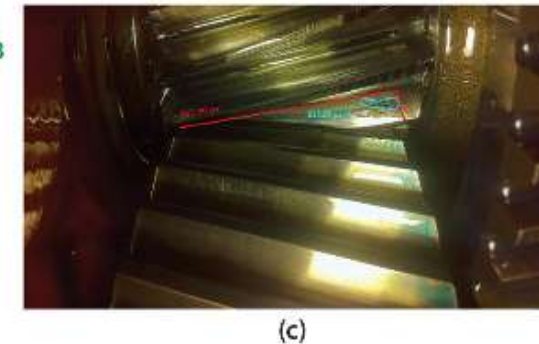
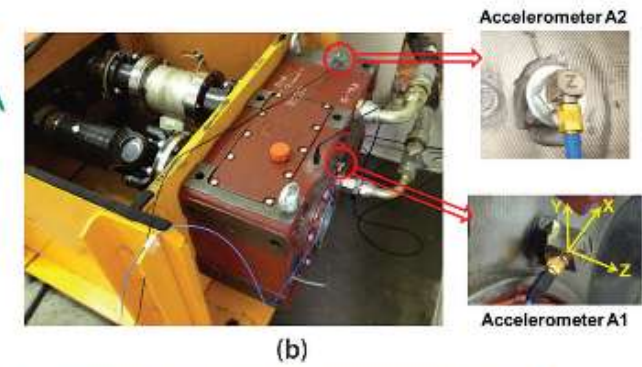
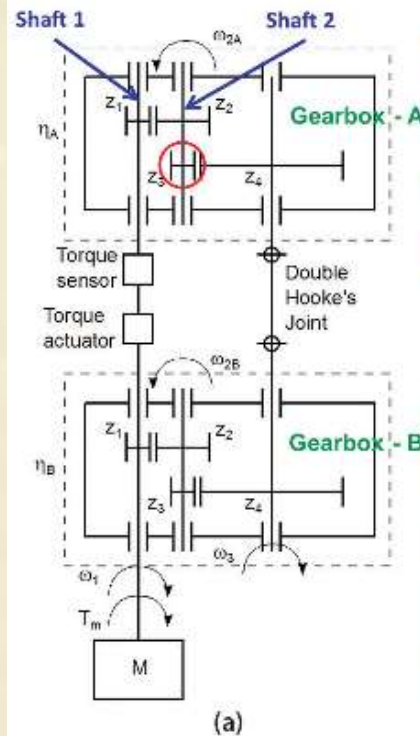
Error on peak max -5.7%  
1000 microstrain vs 943  
microstrain  
(considering average



# Test Rig 1: Test

**Goal:**  
Endurance test

- Results:**
- Overload  $\gg$  100% nominal torque
  - Pitting in reasonable time
  - Vibration data recorded for validating condition monitoring techniques: 5% pitting detectable





## Test Rig 1: Publication

### Authors:

Gelman, Len, N. Harish Chandra, Rafal Kurosz, Francesco Pellicano, Marco Barbieri, Antonio Zippo.

### Title:

Novel spectral kurtosis technology for adaptive vibration condition monitoring of multi-stage gearboxes.

### Publisher:

The British Institute of Non-Destructive Testing

### DOI:

[10.1784/insi.2016.58.8.409](https://doi.org/10.1784/insi.2016.58.8.409)

DOI: 10.1784/insi.2016.58.8.409

VIBRATION ANALYSIS

# Novel spectral kurtosis technology for adaptive vibration condition monitoring of multi-stage gearboxes

L Gelman, N Harish Chandra, R Kurosz, F Pellicano, M Barbieri and A Zippo

*In this paper, the novel wavelet spectral kurtosis (WSK) technique is applied for the early diagnosis of gear tooth faults. Two variants of the wavelet spectral kurtosis technique, called variable resolution WSK and constant resolution WSK, are considered for the diagnosis of pitting gear faults. The gear residual signal, obtained by filtering the gear mesh frequencies, is used as the input to the SK algorithm. The advantages of using the wavelet-based SK techniques when compared to classical Fourier transform (FT)-based SK is confirmed by estimating the toothwise Fisher's criterion of diagnostic features. The final diagnosis decision is made by a three-stage decision-making technique based on the weighted majority rule. The probability of the correct diagnosis is estimated for each SK technique for comparison. An experimental study is presented in detail to test the performance of the wavelet spectral kurtosis techniques and the decision-making technique.*





## Test Rig 2

**Colaboration:**  
cooperation with CNH

**Goal:**  
Vibration measurement

**System:**  
Open loop configuration

**Fully sensorized**

- Torquemeters
- Tone wheel
- Accelerometers (on the shaft)
- Strain gauges
- Slip rings

**Electric power circulation**



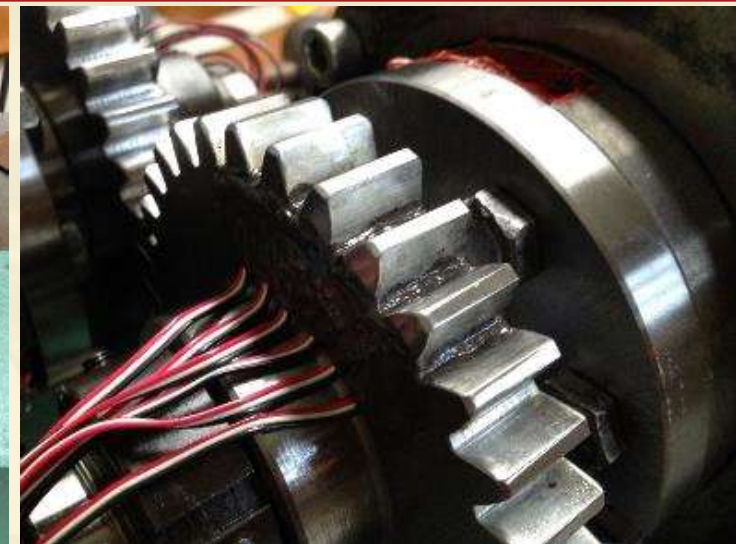
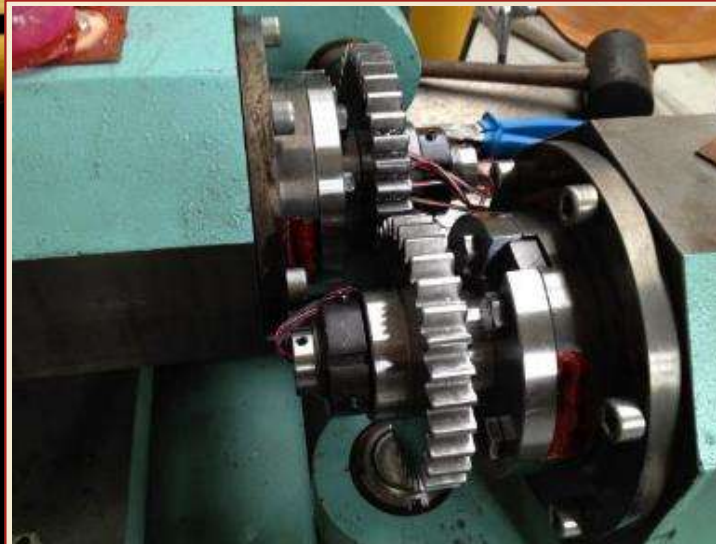
**Motor**

**Gear set**

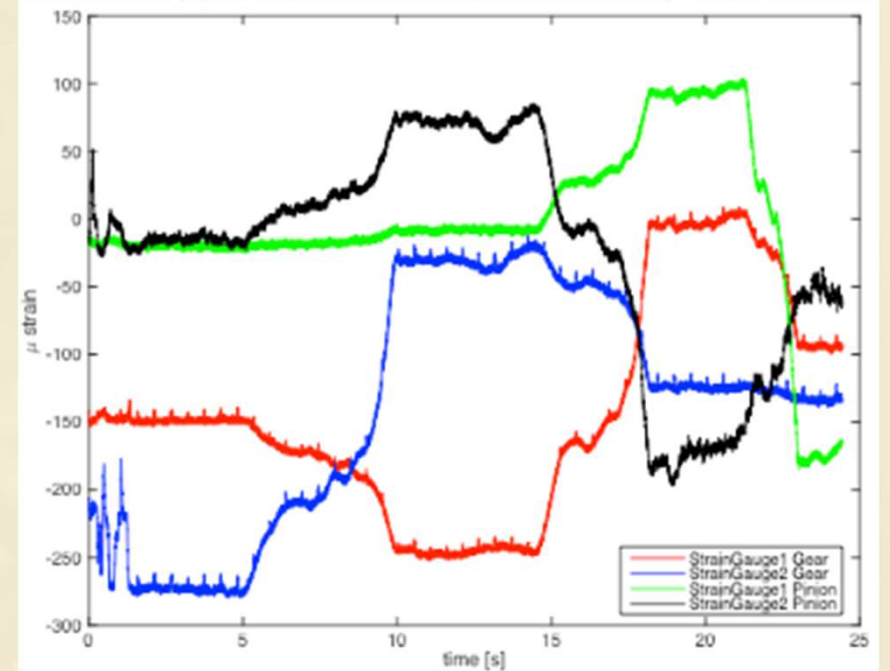
**Brake**

## Test Rig 2:

**Goal:**  
Root stress measurement



Four strain gauges are placed in the root fillet of each gear. The cables are connected to the slip ring through the hollow shaft

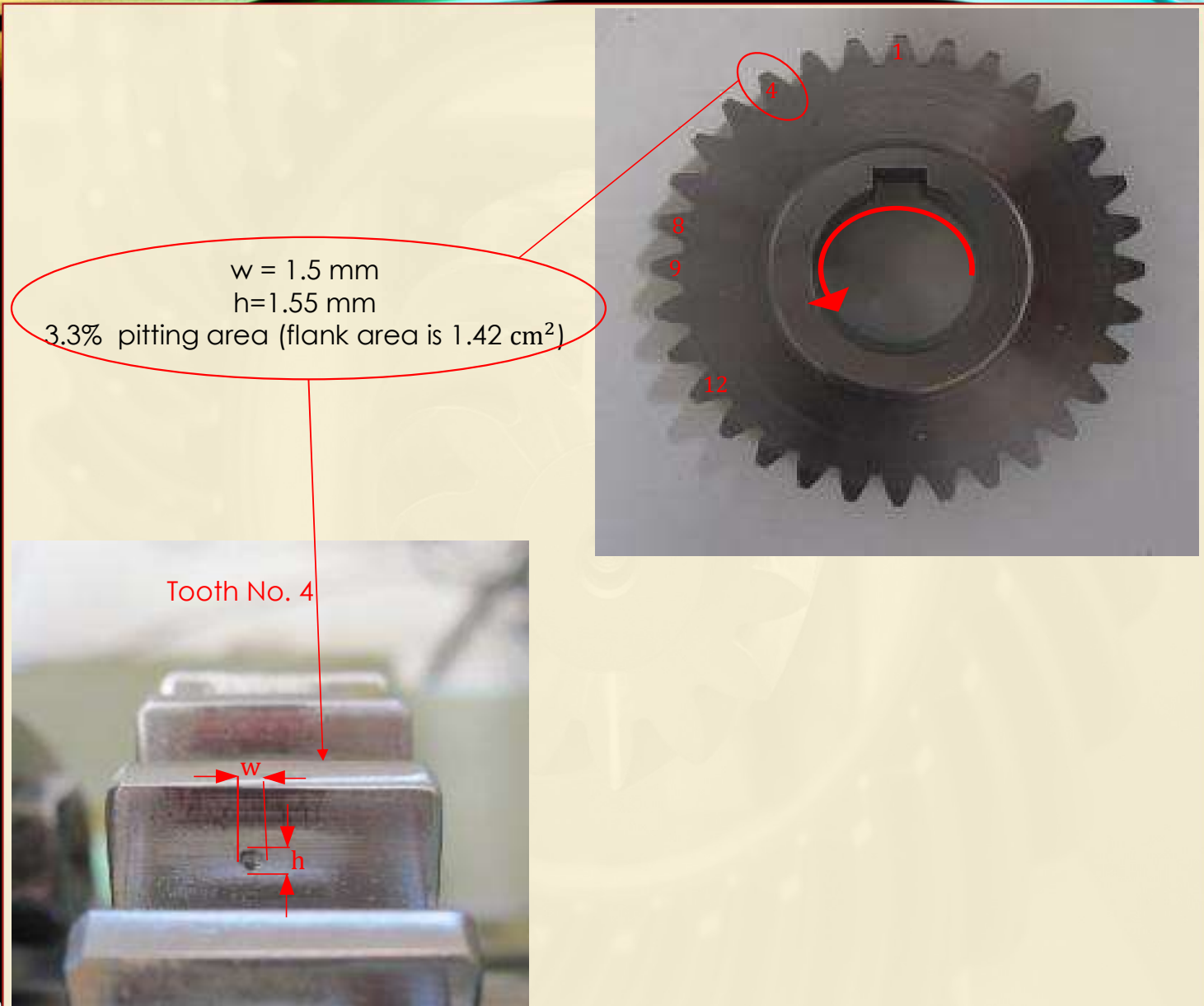


**Collaboration:**  
cooperation with LUKA,  
INDGEAR industrial partner

**Goal:**  
Special design and  
production for fast pitting

**Achievements:**

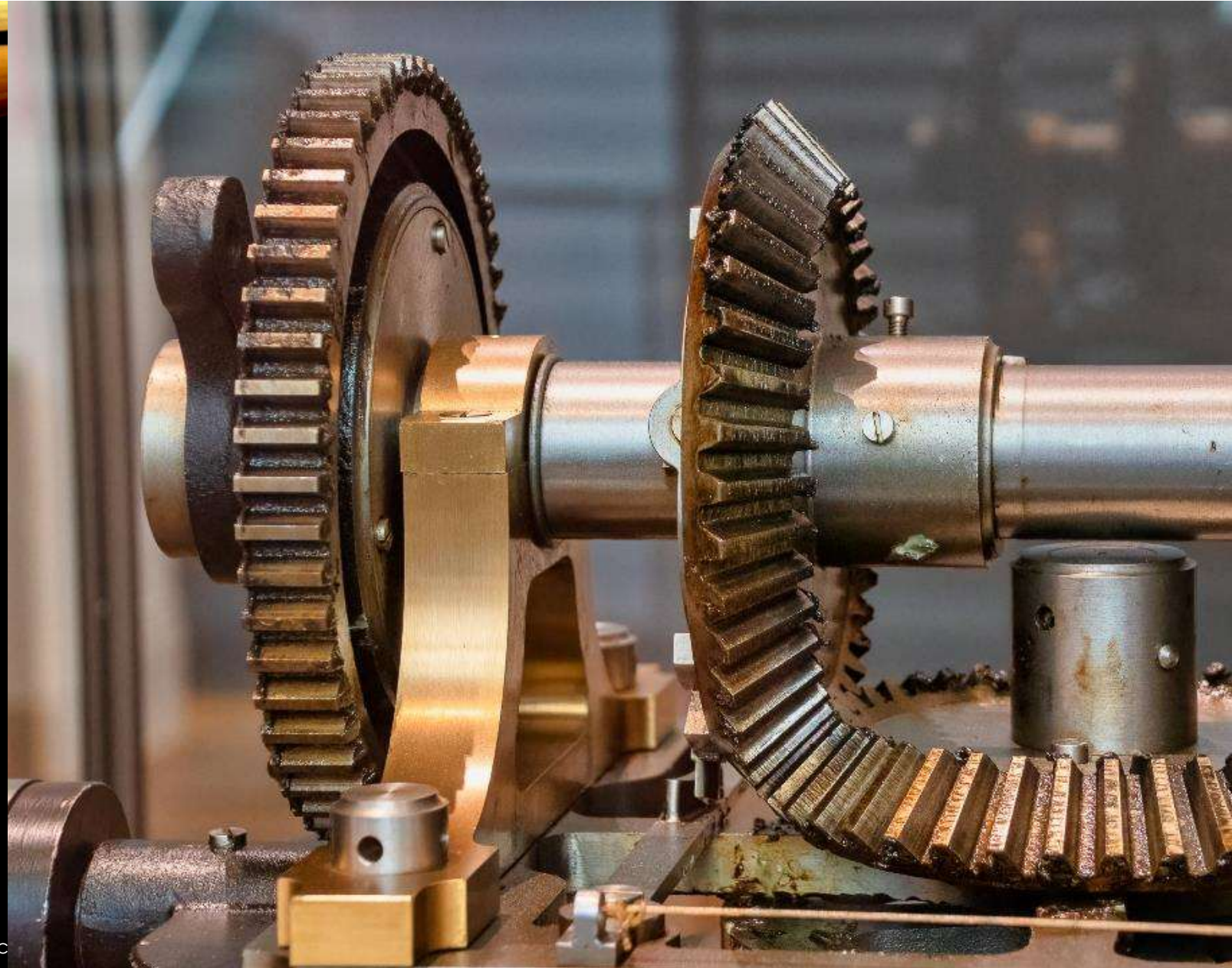
- Accelerated endurance test (pitting)
- Vibration-stress dependencies
- Direct gear stress measurement







# SOFTWARE Simulations



F. Pellicano – A. Zippo - francesco.pellicano@unipi.it

# Type: Planetary gear

**Goal:**  
FEM analysis of planetary gearbox for Robotics

**Software:**  
ANSOL – Transmission3D

## Detailed FEM modelling

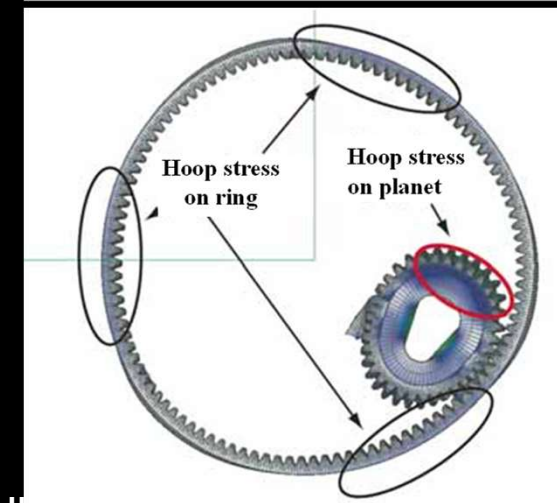
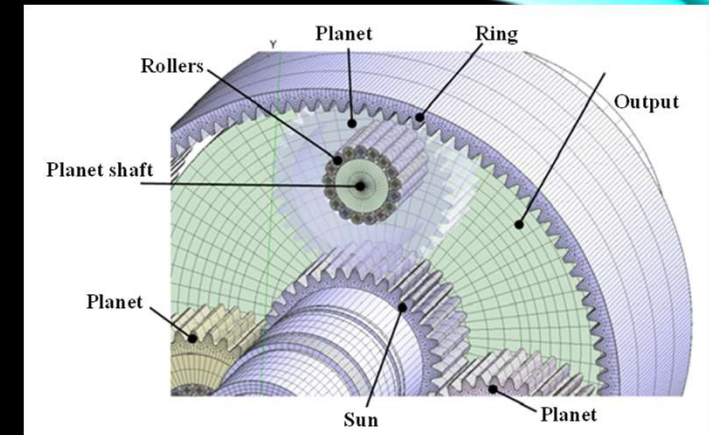
- Housing, carrier and shaft elastic deformation
- Gears
  - Contact analysis
  - Misalignments
- Bearings
  - Cage deformation
  - Rollers: deformation and contact analysis

**Hoop stresses** are generally neglected (or unknown)

- Arise when the ring is thin
  - lightweight gearboxes
  - e.g., helicopter final reduction
- Can cause unexpected failures

**The whole elasticity influences the contact pattern**

- The carrier and shaft deformation induce planets misalignments
- Local stresses depend on:
  - load
  - teeth geometry (crowning)
  - global elasticity





# Type: Helical gear pair

## Goal:

### Hardness effects on gear-pair lifetime:

- Depth of hardening,
- Core hardness,
- Surface hardness.

### Considering safety factors:

- root safety factor, ISO 6336-3,
- flank safety factor, ISO 6336-2,
- safety factor of hardened layer, DNV 41.2,
- safety factor against tooth flank fracture, ISO 6336-4,
- safety factor against scuffing, ISO 13989.

### Software: KISSsoft

**Table 1.** Effect of surface hardness on safety factors for gear-pair No.1

Depth [mm] =2				Core hardness [HRC] =34.6							
Surface hardness	HRC	52.4		54.4		56.4		58.4		60.4	
		Safety factors									
Against tooth flank fracture		1.432	1.450	1.458	1.477	1.481	1.500	1.502	1.521	1.520	1.539

**Table 2.** Effect of core hardness on safety factors for gear-pair No.1

Depth [mm] =3				Surface hardness [HRC] =61							
Core hardness	HRC	29.8		40.8		49.1		55.2		56.3	
		Life time [h]		20,000		20,000		20,000		20,000	
Safety factors	Root	2.406	2.221	2.406	2.221	2.406	2.221	2.406	2.221	2.406	2.221
	Flank	1.481	1.574	1.481	1.574	1.481	1.574	1.481	1.574	1.481	1.574
	hardened layer	1.948		1.948		1.948		1.948		1.948	
	Against tooth flank fracture	1.821	1.842	2.069	2.090	2.125	2.147	1.515	1.534	1.429	1.447
	Against scuffing	3.609		3.609		3.609		3.609		3.609	

**Table 3.** Effect of hardness depth on safety factors for gear-pair No.1

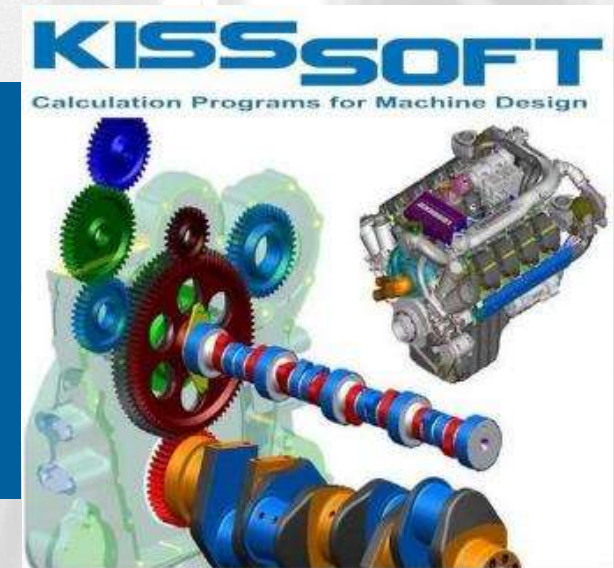
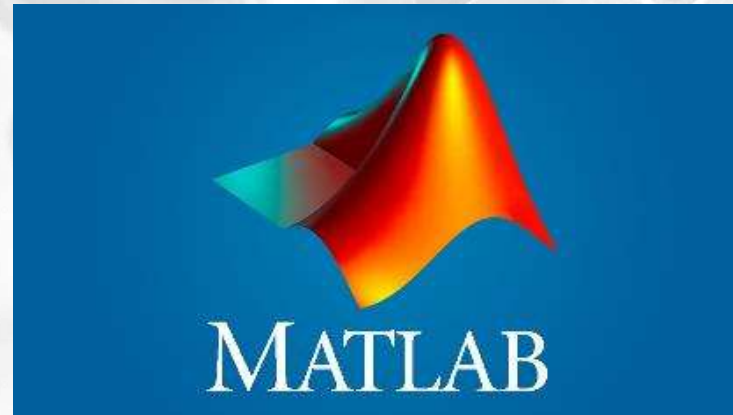
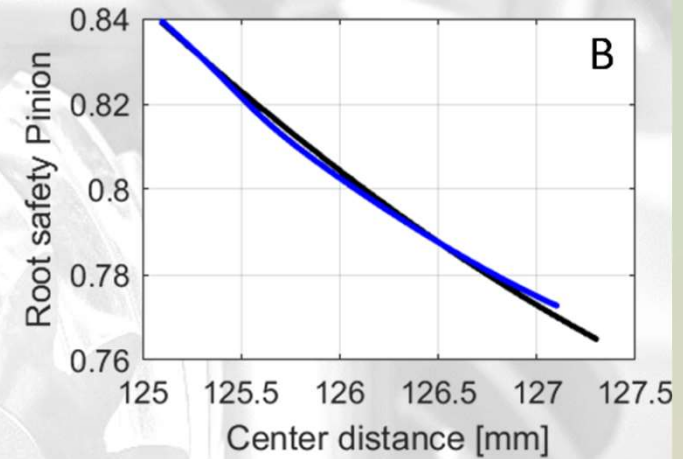
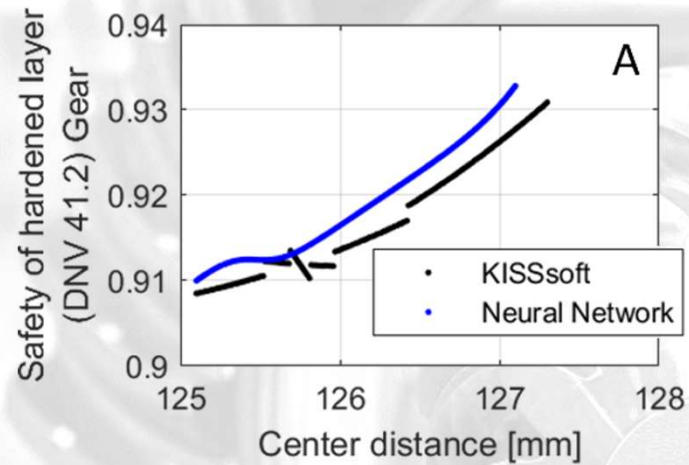
Core hardness [HRC] =34.6				Surface hardness [HRC] =52.4							
Depth [mm]		2-2.1		2.5-2.6		3-3.1		3.5-3.6		4-4.1	
Life time [h]		20,000		20,000		20,000		20,000		20,000	
Safety factor	Root	2.406	2.221	2.406	2.221	2.406	2.221	2.406	2.221	2.406	2.221
	Flank	1.481	1.574	1.481	1.574	1.481	1.574	1.481	1.574	1.481	1.574
	hardened layer	1.696		1.826		1.948		2.064		2.173	
	Against tooth flank fracture	1.432	1.450	1.586	1.606	1.774	1.794	1.727	1.747	1.892	1.913
	Against scuffing	3.609		3.609		3.609		3.609		3.609	

**Type: Helical gear pair**

**Goal:**  
Train the algorithm to predict the life-time

**Software:**  
MATLAB and KISSsoft

## Neural Network Algorithm



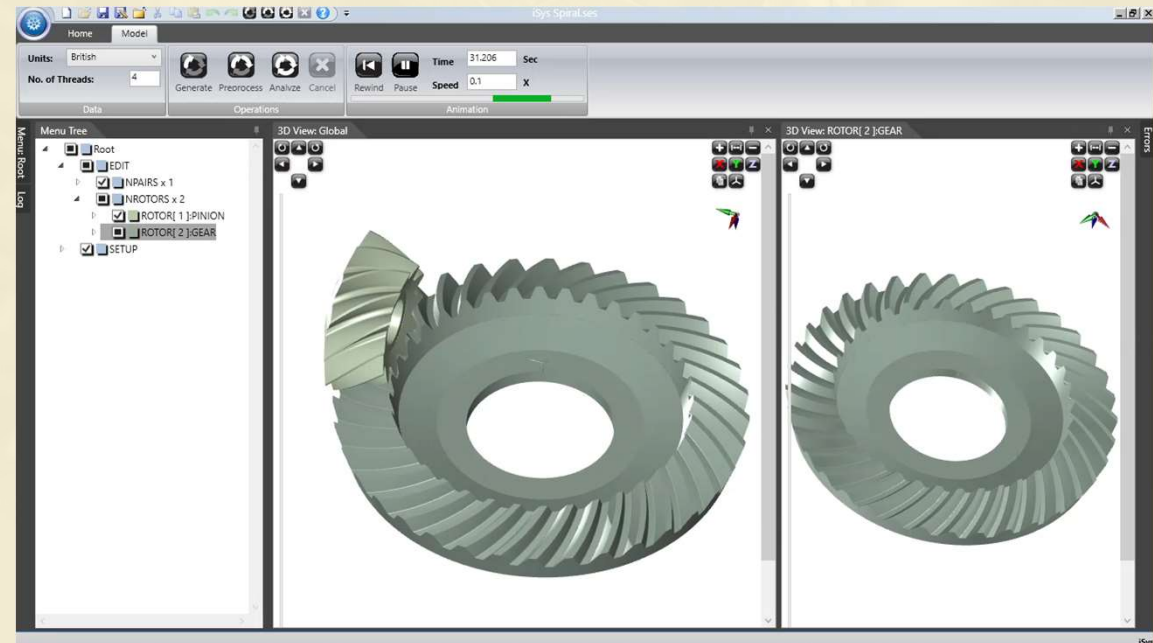
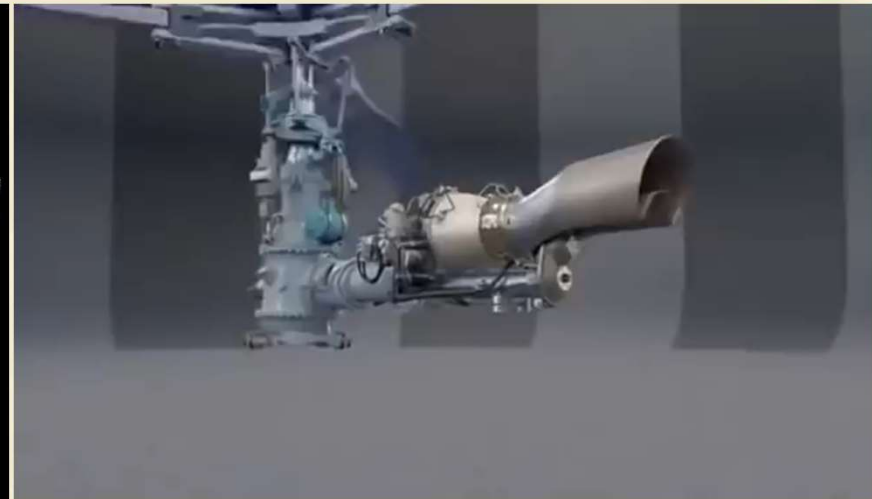
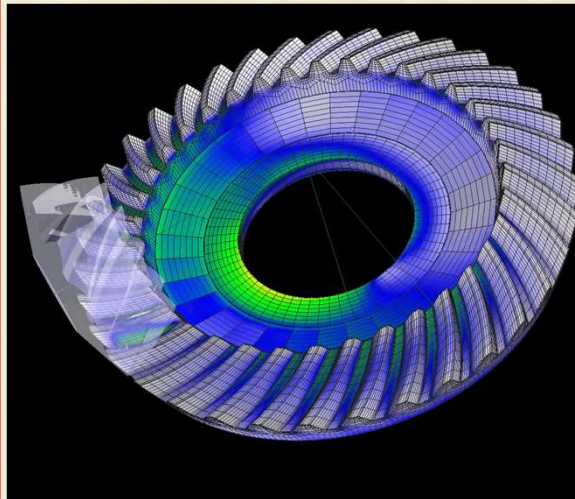
## Type: Spiral Bevel Gear

### Goal:

- Loaded and Unloaded tooth contact analysis (LTCA and UTCA)
- Mesh stiffness calculation

**Application:**  
Helicopter transmission system

**Software:**  
ANSOL – Transmission3D



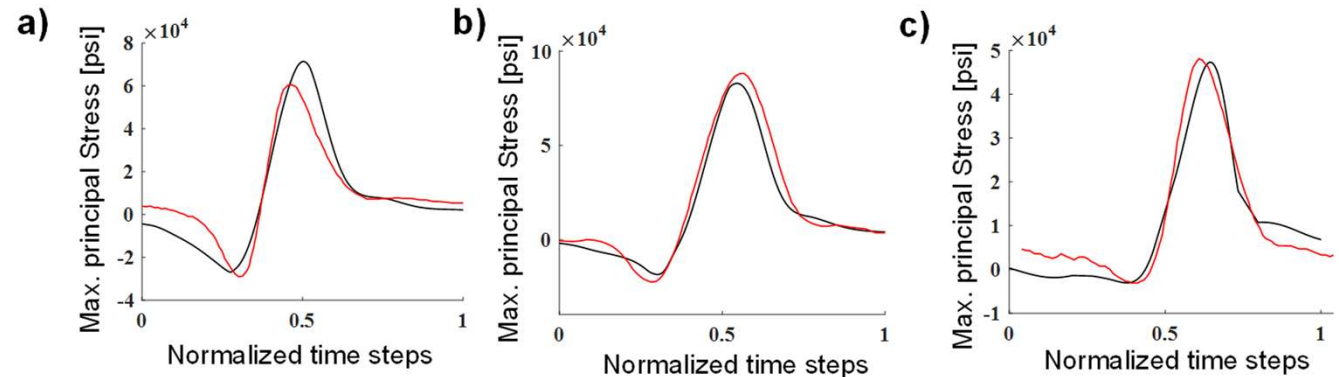
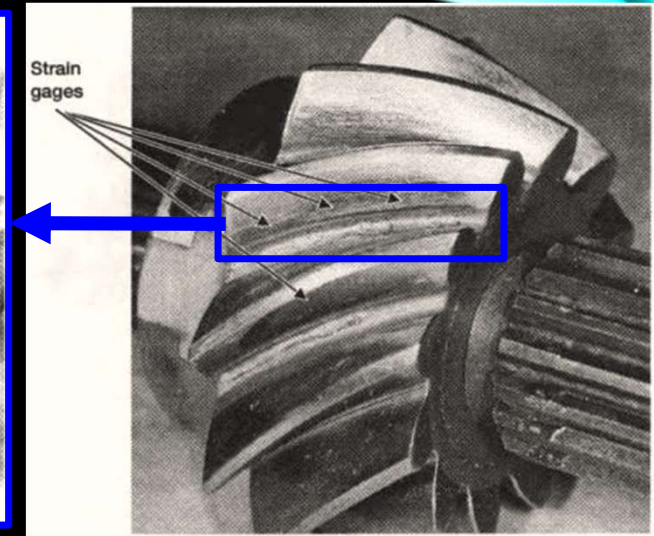
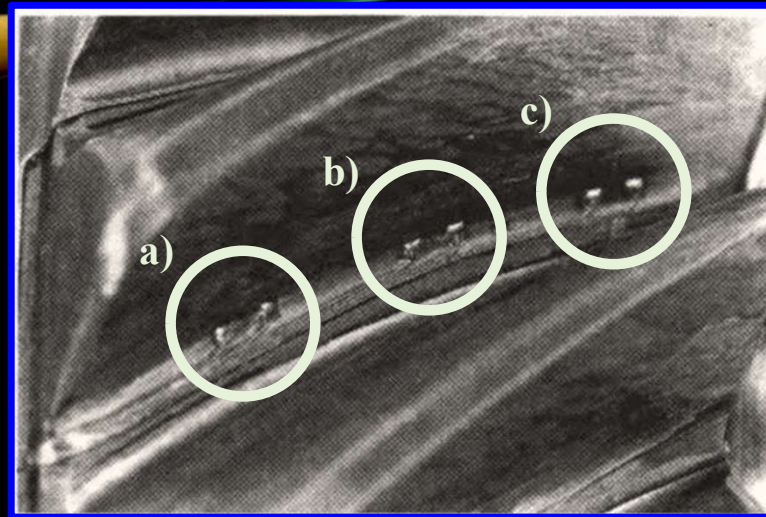


# Type: Spiral Bevel Gear

**Goal:**  
Part 1:  
Validation numerical results

**Comparison with:**  
Experimental data, done by  
NASA group.

**Comparison parameters:**  
Max. principal stress on the  
root of pinion tooth



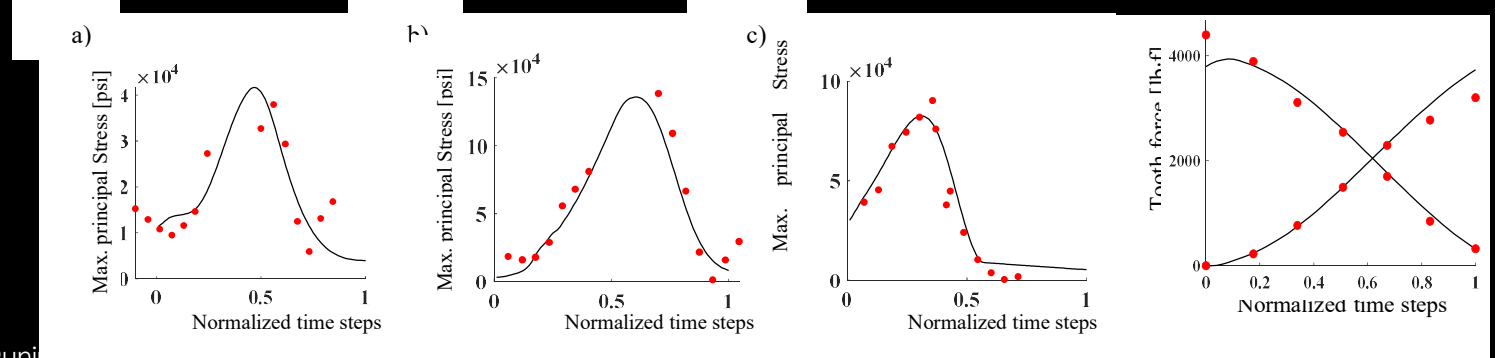
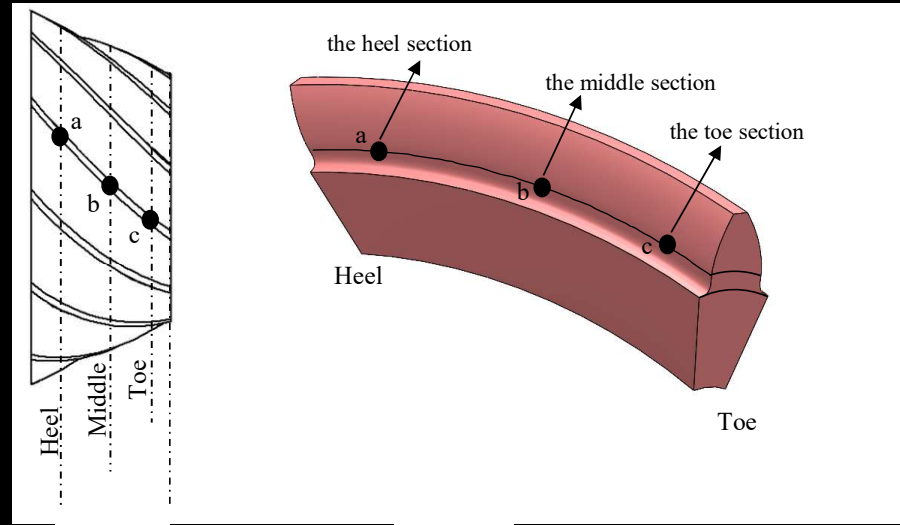
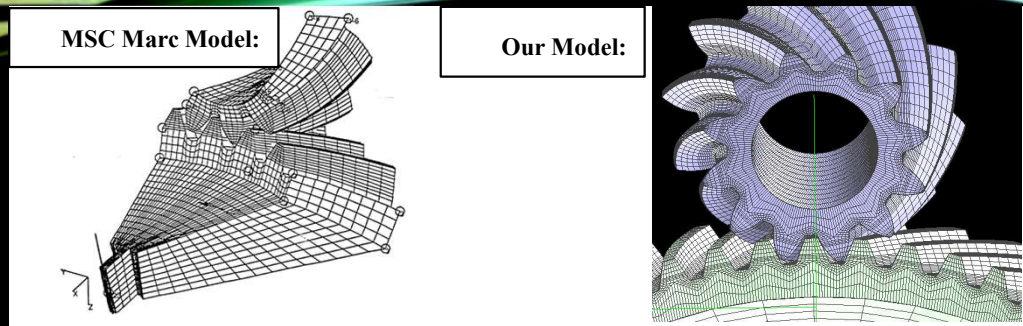
Maximum principal stress on the root of the tooth,  
— FEM (Calyx) simulation, — Experimental results.

# Type: Spiral Bevel Gear

**Goal:**  
**Part 2:**  
**Validation numerical results**

**Comparison with:**  
 Numerical results, FEM (MSC Marc); done by NASA group.

- Comparison parameters:**
- Max. principal stress on the root of pinion tooth
  - Force between two mated teeth





# Type: Spiral Bevel Gear

**Goal:**  
FEM grid sensitivity analysis  
and find the suitable mesh  
size

**Considered parameter:**  
Average rotational deflection of  
gear

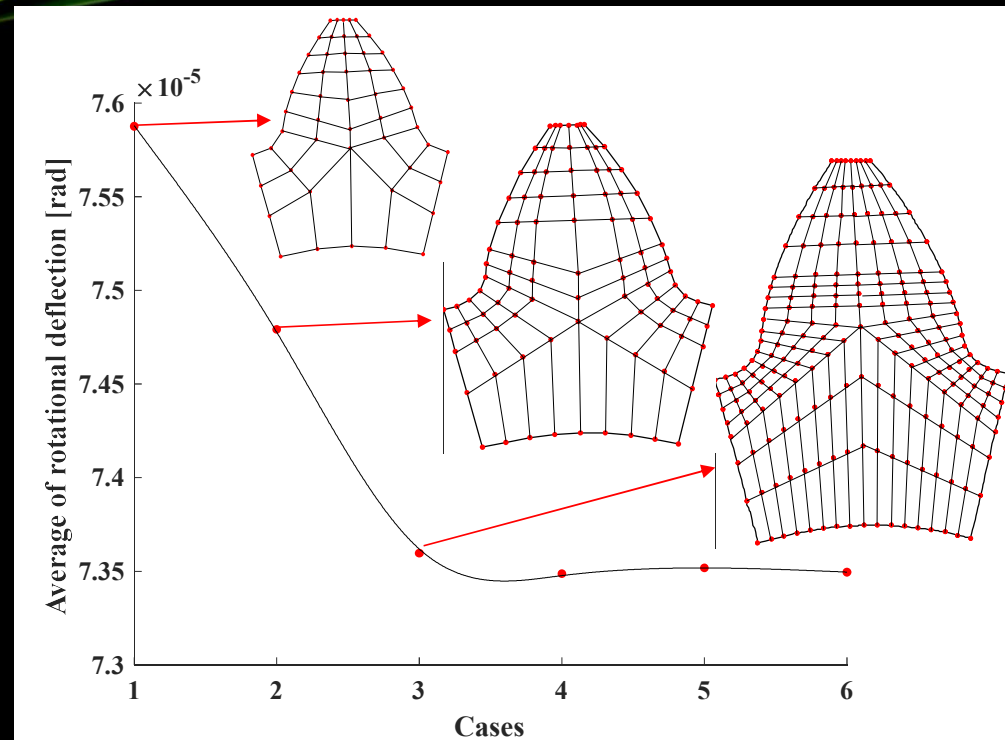


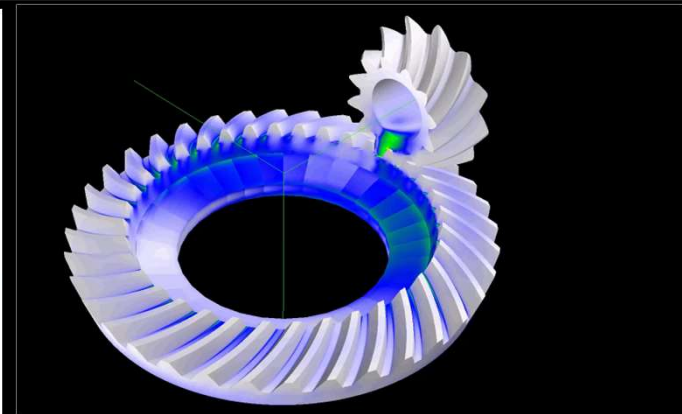
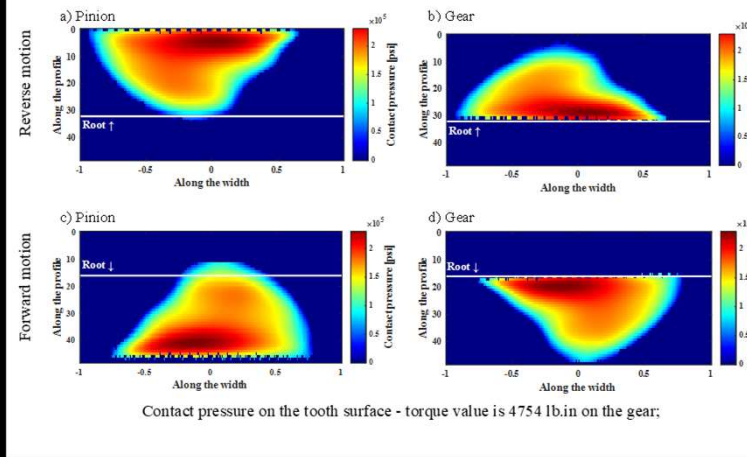
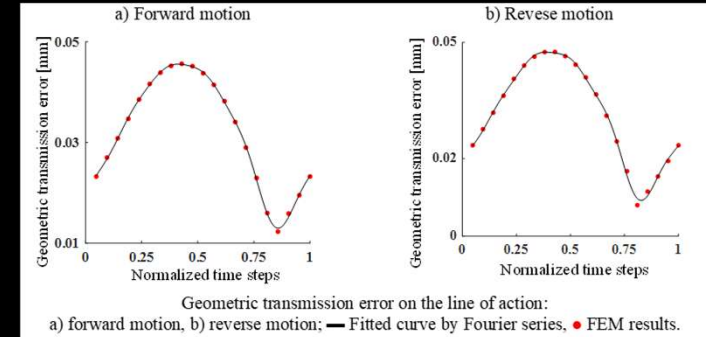
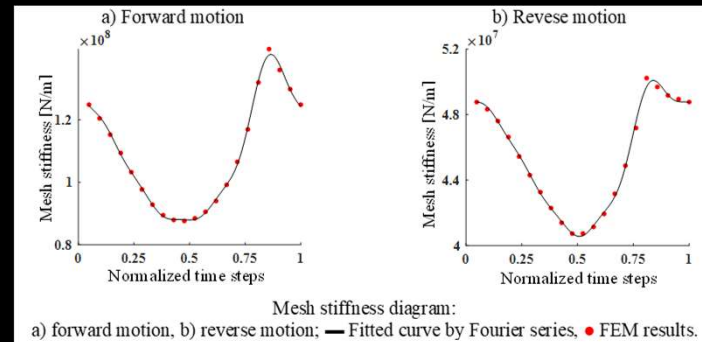
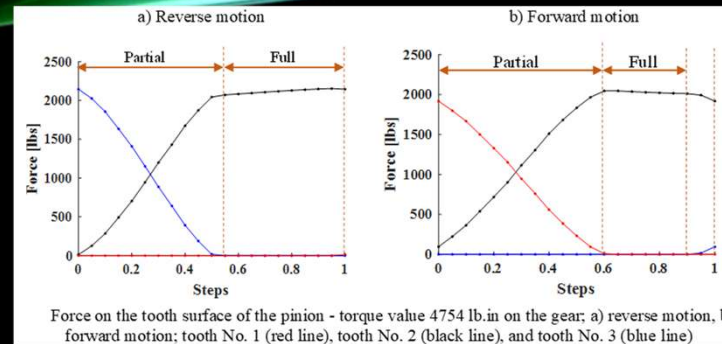
Table 1. Grid sensitivity analysis

Case	1	2	3	4	5	6
Average deflection of gear [ $\mu$ rad]	75.86	74.77	73.60	73.49	73.52	73.49
Number of sections along the tooth width	3	8	13	14	15	16
Number of elements on the tooth section	36	74	176	176	176	176
Computation time [minute]	260	286	398	420	437	454

# Type: Spiral Bevel Gear

## Goal:

- Force on the tooth surface of the pinion
- Contact pressure on the tooth surface
- Mesh stiffness diagram
- Geometric transmission error



# SBG: Publication

## Authors:

Moslem Molaie, Farhad S. Samani, Antonio Zippo, Francesco Pellicano.

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## Spiral Bevel Gears: nonlinear dynamic model based on accurate static stiffness evaluation

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### ARTICLE INFO

#### Keywords:

nonlinear vibration  
spiral bevel gear  
gear mesh stiffness  
forward and reverse motions  
backside contact



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### ABSTRACT

In the present paper non-linear dynamics of a spiral bevel gear pair with backlash are investigated in order to clarify the internal excitations of major importance from the vibration point of view: manufacturing errors in the teeth profile, teeth spacing errors, and elastic deformation of the teeth. In some conditions, like in the case of backside contact, the destructive effect of internal excitations can be intensified leading to complex dynamics; for such reasons here backside contacts and reverse rotation are investigated in detail using a nonlinear time-varying model. The effect of damping is investigated as well. A one-DOF model is developed in order to study the dynamic behavior; the resulting a nonlinear differential equation with time-varying mesh stiffness is solved via numerical integration based on an adaptive step-size implicit Runge-Kutta scheme. The dynamic response of the system is analyzed through time histories, phase portraits, bifurcation diagrams, and Poincaré maps. Results show that for small backlash values, the possibility of backside contact increases. Meanwhile, by increasing the backlash value, the amplitude vibration of the gear rotation rises as well. By comparing the dynamic response of the system with different damping ratios, the results show that higher damping effectively reduces gear vibration resonance, although the probability of unsteady response still exists.





# THE VIBRATION AND POWERTRAIN LAB

Not only Gears

- NVH testing: vibration and noise measurements, modal analysis, materials analysis
- Mechanical and NVH modelling: FEA and Multibody
- Environmental and mechanical tests on shaking table
- Active and passive vibration control
- Problem solving



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# THANK YOU FOR YOUR ATTENTION



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